PROGRAMA DE APOYO DIDÁCTICO

FEBRERO-JULIO 2019

BACHILLERATO TECNOLÓGICO

PROBABILITY AND STATISTICS

Secretaría de Educación
Nuevo León
GOBIERNO DEL ESTADO

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Colegio de Estudios Científicos y Tecnológicos
CECyTE Nuevo León
1993-2018
25 ANIVERSARIO

Probability and Statistics

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UNIT I

Didactic Sequence Num. ___1______ INTRODUCTION TO STATISTICS	
Expected Learning: Know the development of Statistics, through its history, founder, and identify the importance of this in the different fields of application in daily life.	
Competences to Develop:	
Discipline:	M1. Analyze the relationship between two variables of a social or natural to determine or estimate their behavior process.
Generics:	4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 8. Participate and collaborate effectively on diverse teams. 8.3 Assumes congruent with constructive knowledge and skills with which account within different teams.
EXPECTED PRODUCT: Research on the history of statistics, and their applications.	

Opening	Development	Closure
Activity 1: (Self-evaluation) Research collected and graphs made	Activity 2: Underlined reading Activity 3: Reading Summary (Coevaluation)	Activity 4: Team research (Heteroevaluation)

OPENING:

Activity 1. (Individual for self-evaluation)

a) Make a table in your notebook to get the following information of all the classmates in your room.
Name of the subjects of the previous semester:
Age: _____________ Gender: _________
Do you have any failed subjects from the previous semester? Yes __ No___ ¿Wich? _________

Age	Gender	Number of failed subjects	Subject 1	Subject 2	Subject 3

b) With the information collected make a table where you group the data, depending on age, type, number of failed subjects, subject with the highest number of students failed, subject with less number of failed, to make a graph.

c) It is requested on a voluntary basis to present the elaborated works and to make an evaluation of them comparing with their peers and seeing What is it that we lacked to consider?

DEVELOPMENT:

Activity 2. Read carefully the following information and underline what you consider most important.

General Historical Overview of Statistics

Since the beginnings of civilization there have been simple forms of statistics, since graphic representations and other symbols were used in skins, rocks, wooden sticks and walls of caves to count the number of people, animals or certain things.

Towards the year 3000 BC the Babylonians already used small clay tablets to collect data in tables on agricultural production and of the goods sold or exchanged by barter. The Egyptians analyzed the data of the population and the income of the country long before constructing the pyramids in the XXXI century a.C.

The Biblical Books of *Numbers and Chronicles* include, in some places, statistical works. The first contains two censuses of the population of Israel and the second describes the material well-being of the various Jewish tribes.

In China there were similar numerical records before the year 2000 A.C. The classical Greeks carried out censuses whose information was used around the year 594 BC. to collect taxes.
The Roman Empire was the first government to compile a large amount of data on the population, area and income of all the territories under its control.

The word ***statistic*** comes from an Italian ***statist*** voice, which means statesman. The coined Gottfried Achenwall (1719-1772), and Dr. Zimmerman introduced it in England. Its use was reported by Sir John Sinclair in his work entitled *Statistica lAccount of Scotland* (1791-1799).
It is considered that ***the founder of the statistic was Gottfried Achenwall***, in 1748, when conducting population studies that were later supplemented with the theory of probabilities, which were jointly applied to update social aspects, such as birth, crime, mortality, education, diseases, etc.

In 1829 the Belgian statesman ***Adolfo Quetelet***, was the first to apply statistical methods in the investigation of educational and social problems; contributed in the elaboration of the first official European censuses, as well as in the development of the equality and similarity of statistical data between nations; which is why he is considered ***the father of modern statistics.***

Nowadays, statistics have become an effective method to accurately describe the values of economic, political, social, psychological, biological and physical *data*, and serve as a tool to relate and analyze such data. The work of the statistical expert is not just about gathering and tabulating the data, but above all the process of interpreting that information.

Definition of Statistics.

The most widespread acceptance of the word **Statistics** is closely linked to the counting and enumeration activities traditionally carried out by the State with different purposes: to know the number of inhabitants of a country or region; record crop volumes of agricultural production; have a standard for collecting taxes, etc. Under these conditions it is not surprising that it is associated with large lists of numbers whose extension makes them incomprehensible.

In this way, the term **"statistics"** has different meanings:
1) Numerical information,
2) Method to obtain, organize, present and describe large amounts of data,
3) Method to make decisions and
4) Study area, that is, a discipline.
A definition of statistics for operational rather than totalizing purposes could be the following:

STATISTICS	It is a branch of applied mathematics, whose purpose is to collect, organize, summarize, present and analyze numerical data related to a set of individuals or observations that allow us to draw valid conclusions and make logical decisions based on those analyzes.

Applications of Statistics

The systematization and improvement of statistics at present, allows you to intervene in all fields and activities of the human being, as an indispensable instrument for making decisions that allow us to structure this society, which is increasingly more complex.

In these fields, a large number of different data are collected, many of them come from measuring or counting instruments, so the knowledge of statistics is essential in the interpretation and analysis of data, supporting the researcher in his studies, allowing to find the most useful and practical applications. Among the fields in which the statistics are applied, are:

- **Government agencies**, both at the federal and state levels, require statistical data for the future, for example, demographic trends, fluctuations in the stock market and the index of industrial production, among other aspects.
- In **education and psychology**, statistics plays an important role; For example, an educator can determine if there is a relationship between the achievement of test scores and the average score points in a certain category of students and in this way make predictions about them.
- In **biological sciences**; for example, in agriculture statistical methods are used to determine the effects of certain kinds of seeds, insecticides and fertilizers in the fields; to determine the sowing periods and the rain calendars.
- In **medicine**, in order to establish the possible side effects or the effectiveness of medicines and to provide better methods in order to control the spread of contagious diseases.
- In **physics**, to obtain data and test hypotheses.
- In **Engineering**, statistical principles are applied in total quality control.
- In **the industrial administration** for human, material, economic, time and movement resources
- In **finance** for real estate, investments and stock market.
- In commerce for market studies and supply and demand analysis.

Most of the research carried out in the various disciplines of science, these include in their observations numerical values (data), so, when making measurements or counts, it is necessary to have a help-assistant in the presentation, analysis and interpretation of the data this help is *statistics*.

Activity 3. Make a summary of the reading based on what is underlined, and be reviewed by the teacher.

CLOSURE: (Team research, for coevaluation)

Activity 4. They meet in teams to share what they have learned in reading, in addition to conducting an investigation of the importance of the application of statistics in daily life (newspapers, internet, magazines, etc.) and presenting it to their classmates through of any of the following techniques: collage, conceptual map, mental map, synoptic chart, etc.

UNIT I

<table>
<tr><td colspan="2">DIDACTIC SEQUENCE Num. ___2____ FUNDAMENTAL ELEMENTS</td></tr>
<tr><td colspan="2">Expected Learning: Use your own language for situations that need to be studied with elements of statistics and probability.</td></tr>
<tr><td colspan="2">Competences to develop:</td></tr>
<tr><td>Discipline:</td><td>M2 Formula and solve mathematical problems, applying different approaches.
M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations.
M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies.
M6. It quantifies, represents and contrasts experimentally or mathematically the magnitudes of space and the physical properties of the objects that surround it.
M7. Choose a deterministic or a random approach to the study of a process or phenomenon, and argue its relevance.</td></tr>
<tr><td>Generics:</td><td>1. Know and value yourself and address problems and challenges taking into account the objectives pursued.
1.1 Faces the difficulties that arise and is aware of their values, strengths and weaknesses.
1.6 Manage available resources taking into account the constraints to achieve your goals.
4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools.
4.1 Express ideas and concepts through linguistic, mathematical or graphic representations.
4.5 Manages information and communication technologies to obtain information and express ideas.
5. Develops innovations and proposes solutions to problems based on established methods.
5.3 Identify the systems and rules or core principles that underlie a series of phenomena.
7. Learn by initiative and self-interest throughout life.
7.2 Identify the activities that result in less and greater interest and difficulty, recognizing and controlling their reactions to challenges and obstacles.</td></tr>
<tr><td colspan="2">LEARNING PRODUCTS: Prepare a table with examples of population, sample and observation unit.

Openig   Development   Closure</td></tr>
</table>

Activity 1. (Self-evaluation) Questions	Activity 2. Underlined reading Activity 3. Conceptual map (Heteroevaluation)	Activity 4 and 5. Individual work (Heteroevaluation)

OPENING: (Individual self-evaluation)

Activity 1. Read carefully the following information to answer the questions that are presented below.

A high school has the following students.

Semester	**Men**	**Women**	**Total**
First	200	225	425
Third	100	115	215
Fifth	95	105	200

Of the total of women, 45 of them are blond and the rest of them are brown-haired. Of the men 50 of them like to play basketball, 30 of them like boxing and the rest of them prefer soccer.

Mention the amount of population that the school has. ____________________________ Mention the *number* of women _____ and men ________ that exist in that school. Mention the subgroups that are being talked about ______________________________

Mention some of the *qualities* that are mentioned. ___________________________

Based on the answers given, answer freely in your notebook what you understand about the following concepts:

1. What does data mean? 2. What is a data collection? 3. What is a variable? 4. What do you understand by a population? 5.What is sample? 6. What do you understand by qualitative? 7. What do you understand by quantitative?

Share the answers obtained with the rest of the group and complement those that are needed or those that you have not made.

DEVELOPMENT:

Activity 2. Individually read the following topics, do not forget to underline what you consider most important.

Division of Statistics. The statistic is divided for its study into two branches, which are:

DESCRIPTIVE STATISTICS	It´s the part of statistics, which includes the obtaining, presentation and description of numerical data, without pretending to obtain conclusions or inferences of a more general type.

Descriptive statistics is also called ***Deductive statistics***. Usually they represented by tables, graphs and indexes.

INFERENTIAL STATISTICS	It´s the part of statistics that deals with the techniques for making decisions based on partial or incomplete information obtained through descriptive techniques.

Inferential statistics is also called ***Inductive statistics***, based on the results obtained from the analysis of a sample of the population, it infers, induces or establishes the laws of behavior of the population to which it belongs.

Basic concepts of statistics. Population and sample.

The first concern about a data set is whether it can be considered as all possible data or only a part of a larger set. This is of great importance, and not making a clear distinction has produced errors in the way of thinking and an ambiguous explanation in some writings. Therefore, it is essential to establish the following basic concepts:

POPULATION or UNIVERSE	It´s defined as the source of observations or measurements that describe in detail a set of individuals or objects. That is, it is the totality of possible measurements (or counts) and observations of a given situation.

If the population is abundant, it is sometimes impossible or unusual to observe the totality of the elements that make it up, so it is recommended to analyze a representative part of said set and which we call ***sample.***

SAMPLE	It´s defined as a subset of the study population. A sample is also defined as the set of observations that represent the totality of the characteristics to be examined of a population.

In general, it´s about using the sample information to make inferences about a population. For this reason it is particularly important to define the population that is studied and obtain a representative sample of the defined population.

OBSERVATION UNIT	It´s a single member of the study population.

Observations are the raw material with which researchers work. In order for statistics to be applied to these observations, they must be in numerical form. In the improvement of crops, the numbers may well be yields per plot; in medical research, recovery times may be under various treatments; in the industry, there may be a number of defects in several batches of an article produced in an assembly line.

Such numbers constitute data and their common characteristic is variability or variation.
The following sets constitute an example of a population, sample and an observation unit:

Population		Sample		Observation unit
• The **ages** of the **students** of the CECYTE-Nuevo León.		• The **ages** of the **male** students of the CECYTE-Nuevo León.		• **One** of the **ages** of male students.
• The **production of bottles** processed in the company Vidriera de Monterrey during the month of May.		• The **production** of bottles processed in the company **Vidriera** de Monterrey in the morning shift on May 15.		• **One** of the **bottles** produced.
• The number of **manufacturing companies** in the city of Monterrey, N. L.		• The number of **electronic products** manufacturing **companies** in the city of Monterrey, N. L.		• **A manufacturing company** of electronic products.

Variables

Proposals such as "Maria is blonde", or "He weighs more than 80 Kg." Are common and informative. They refer to characteristics that are not constant, but vary from one person to another and that serve to distinguish or describe. The characteristic that present variability or variation are called variables.

VARIABLES	It´s a characteristic of each individual element of a population or a sample. They are amounts to which an unlimited number of values are assigned.

The age of a student when entering the university, the color of his hair, his height, his weight, etc., are variables of response. The value of the variable will be the measure of the characteristic of interest.
The variables can be ***quantitative*** or ***qualitative.***

QUANTITATIVE VARIABLE	It´s the one for which the resulting observations can be measured because they have an order or natural range.

The observations on quantitative variables allow to classify them in: **Continuous and discrete or discontinuous**. In most cases, the distinction between both variables is as easy as deciding whether the data result from a "measurement" or a "count". So their definitions are:

CONTINUOUS QUANTITATIVE VARIABLE	It´s the one that can present any value between two given values, that is, it has a lower limit and an upper limit.

For example, Juan says he weighs 78 kilograms, in "round" numbers; however, we can only be sure that Juan's weight is between 77.5 and 78.5 kg. or any value within the range 77.5 to 78.5. Other examples are:

- The time traveled from home to school.
- The centimeters of rainfall in a region during the year.
- Duration of the light bars.
- The liters of gasoline sold on several days of the month.

QUANTITATIVE VARIABLE DISCRETE or DISCONTINUOUS	It´s the one for which the possible values can´t be observed in a continuous scale due to the existence of spaces between these possible values.

Often discrete observations are whole numbers because they come from counts. Examples are the number of petals of a flower, the number of families residing in an apple or the number of insects trapped in a network.

Other examples are:

- The number of books on a shelf.
- The number of children born at different times of the day.
- The number of CECyTE students of the Linares campus.
- The number of goals scored by the Mexican team in the 2006 World Cup.
- The number of tires in a car.

QUALITATIVE VARIABLE	It´s the one for which it´s not possible to make numerical measurements.

The qualitative aspect of a variable comes from the concept of classified topics; This kind of information is called "attribute data".

For example: If a person went to a parking lot and there he began to classify the vehicles according to their color (to any other similar characteristic), he would find that the response variable assigned to each car is blue, yellow or whatever color it has, I would get attributes. Color is a quality that in this case has no numerical measure.

Therefore, in this type of variable the observations can not be ordered or measured in a significant way, they can only be classified and enumerated.

Data collection.

DATA	They comprise the set of values assigned to the variable for each element belonging to the sample.

The data are situations or events that are represented numerically and that are part of our daily life, sometimes, and others are in books, because they have been collected by other people before.

The types of data can be:

- ***ORIGINALS*** Are those that are collected by ourselves, that is, that are verifiable in a rigorous way.

• ***INDIRECT*** These are those that are compiled from encyclopedias, record books, recorded audio and video events, etc.

For the statistics to be accurate and true, the data collection must be careful and precise, making use of the means, resources and procedures that objectively facilitate its collection.

For example:

1.- Through questionnaires and interviews conducted by competent and professional people, to give rise to the original data.

2.- Through consultations in original and faithful sources, to give rise to indirect data.

Activity 3.
Make a conceptual map in your notebook considering the information underlined in the previous reading.

CLOSURE:
Activity 4. Do each of the following exercises individually in your notebook.

a) Consider the table shown above where examples of population, sample and observation unit are presented, make 5 examples.

Population		**Sample**		**Observation unit**

b) Read the following statements and analyze the concepts that are asked and write on the lines if it is a qualitative or quantitative characteristic.

Statement	Quality or Quantity
c) Marcos has blue eyes.	____________________
d) The group IVA students that use glasses	____________________
e) 15 year old women who are 1.60cm tall	____________________
f) Susana is very nice	____________________
g) El equipo de fútbol soccer tiene 11 integrantes	____________________

Activity 5. Answer the exercises presented below.
Indicates the type of variables that correspond to each of the following statements **(qualitative or quantitative)**

Statement	**Type of variable**
Favorite food	
Profession that you like	
Number of goals scored by your favorite soccer team last season.	
Number of students of your Institution	
The color of your classmates' eyes	
Intellectual coefficient of your classmates.	

The following variables indicate which are discrete and which are continuous.

Statement	**Type of variable**

Number of shares sold each day in the Stock Exchange	
Temperatures recorded every hour in an observatory.	
Duration of a car.	
The diameter of the wheels of several cars.	
Number of children of 50 families.	
Annual census of the Spanish.	

Classify the following **variables** in **qualitative** and **quantitative** discrete or **continuous**.

Statement	**Type of variable**
The nationality of a person.	
Number of liters of water contained in a deposit.	
Number of books in a Library	
Sum of points taken at the launch of a pair of dice.	
The profession of a person.	
The area of the different tiles of a building.	

UNIT I

Didactic Sequence Num. ___3______	DESCRIPTIVE STATISTICS
Expected Learning: Organizes information as part of statistics for the study of probability.	
Competences to develop:	
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M6. It quantifies, represents and contrasts experimentally or mathematically the magnitudes of space and the physical properties of the objects that surround it. M7. Choose a deterministic or a random approach to the study of a process or phenomenon, and argue its relevance.
Generics:	1.1 Faces the difficulties that arise and is aware of their values, strengths and weaknesses. 1.6 Manage available resources taking into account the constraints to achieve your goals. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 4.5 Manages information and communication technologies to obtain information and express ideas. 5.3 Identify the systems and rules or core principles that underlie a series of phenomena. 7.2 Identify the activities that result in less and greater interest and difficulty, recognizing and controlling their reactions to challenges and obstacles.
LEARNING PRODUCTS: As a team, organize a database and deal with a Frequency Distribution Table.	

Opening	Development	Closure
Activity 1: Questions regarding the receipt of light. (Self-evaluation)	**Activity 2:** Summary (Heteroevaluation) **Activity 3:** Procedure of the realization of the example (Heteroevaluation)	**Activity 4:** Team research (Co-evaluation) **Activity 5:** Problem solving by means of tables (Co-evaluation)

OPENING:

Activity 1. Look at the following image of a light receipt, and individually answer the following questions in your notebook, then discuss them in a group guided by your teacher.

Questions:

In teams observe the receipt and answer:

- Identify the variable used in the receipt history.
- Indicate the major data and the minor data.
- Describe the type of graph presented.
- What are the data required to make the graph.

Recognize if the receipt of light is related to the Statistics.

DEVELOPMENT:

Activity 2. Read carefully the following information highlights the most important and makes a summary.

Descriptive statistics

The ***descriptive statistics*** or data processing includes the obtaining, presentation and description of numerical data; that is, consider how the data can be presented visually, and highlight certain characteristics so that they are more objective and useful; therefore, it investigates the methods and procedures, and establishes rules so that data management is efficient, so that the information presented is reliable, express in simple language the contents so that the greatest number of people understand it and can establish comparisons and obtain conclusions.

Statistical research is the operation that refers to the collection of information about a population or group of individuals, measures or objects that have a common characteristic. This includes: Indicating the element of the population that originates the information (research unit), can be: an industry, a home, the person, etc.

En todo caso, la unidad debe ser en su definición medible y fácilmente identificable.

Cite: How should it be? When is it? Where is the place of the What is researched? perform?
will perform? investigation?

- The collection of the information includes: ordering it, filtering it eliminating possible errors and analyzing it, applying the statistical methods and norms.
- The publication of information either for own or for others' use.

When obtaining the information resulting from a statistical investigation, which may have been done, for example, in medicine, to study the behavior of patients subject to a specific treatment; in education, the essays aimed at studying changes in attitude and learning of students subjected to certain educational processes; in agriculture, aimed at measuring the effect of an insecticide under certain conditions that vary under the control of the researcher, etcetera. Next, it is necessary to choose the way to organize it for analysis or publication.

The methods to describe datasets are: ***a) Tabular and graphical. b) Numerical.***

In the scientific, business or public administration reports, as well as in magazines and newspapers, the data are presented by means of tables.

In general, when studying a particular problem, a large amount of data is collected; these data must be organized systematically so that their presentation is useful and meaningful.

This organization can be ordered in ascending or descending order. Whatever the origin, it is an arrangement.

Distribution of data frequencies. The data collection consists of collecting a series of data, which must be organized, based on a numerical order and in subgroups, according to their common characteristics.

If we order the data in ascending order and place a mark in front of them (it can be a vertical line, an asterisk, period, etc.) each time it is presented; the number of marks represents the ***frequency*** with which each piece appears.

The presentation form that results from organizing the data is called frequency ***tables or frequency distribution tables.***

Example:
The following data set represents the final math (algebra) scores of 40 students.

95	**70**	**75**	**69**	**100**	**100**	**74**	**45**
55	**55**	**56**	**77**	**80**	**68**	**77**	**78**
74	**66**	**96**	**65**	**87**	**98**	**70**	**75**
60	**70**	**65**	**52**	**82**	**83**	**92**	**89**
58	**85**	**95**	**70**	**49**	**92**	**64**	**93**

1. Sort the data in ascending order

45	**56**	**65**	**70**	**75**	**80**	**89**	**95**
49	**58**	**66**	**70**	**75**	**82**	**92**	**96**
52	**60**	**68**	**70**	**77**	**83**	**92**	**98**
55	**64**	**69**	**74**	**77**	**85**	**93**	**100**
55	**65**	**70**	**74**	**78**	**87**	**95**	**100**

2.- Considering the previous table, mark with an asterisk * the number of times each quantity appears, you get the following table:

45	*	**58**	*	**68**	*	**77**	**	**85**	*	**95**	**
49	*	**60**	*	**69**	*	**78**	*	**87**	*	**96**	*
52	*	**64**	*	**70**	****	**80**	*	**89**	*	**98**	*
55	**	**65**	**	**74**	**	**82**	*	**92**	**	**100**	**
56	*	**66**	*	**75**	**	**83**	*	**93**	*		

When you have a large amount of data, it is recommended to distribute them in ***classes or categories*** and determine precisely the *number of data* belonging to each class, which is called ***"class frequency".***

Therefore, a **table or frequency distribution** is one in which we organize the data in classes, that is, in groups of values that describe a characteristic of the data (categories or intervals) and that show its frequency; that is, the number of observations (elements) that belong to each class.

There is no general rule for establishing the number of intervals (classes). However, it is recommended that it be from 5 to 15, since it is rarely useful to use less than 5 and more than 15. In addition, we have the ***Sturges* formula** for this purpose, which gives acceptable results:

$$K = 1 + 3.3 \log n$$

Where: ***K*** represents the number of intervals or classes and ***N*** is the total number of data.

When calculating K with the Sturges formula, it is usually not a whole number, so the following criterion is established:

• If the decimal part of the value of K is greater than or equal to 0.5, K takes the value of the next higher integer.

• If the decimal part of the value of K is less than 0.5, K takes the value of the next nearest integer. Let us obtain the value of K, that is, the number of intervals, using the formula of Sturges in the example developed above.

$$K = 1 + 3.3 \log 40 = 1 + 3.3(1.60) = 1 + 5.28 = 6.28 \quad \text{So that } K = 6$$

In all data sets it can be observed that there is a larger data and a smaller data, the difference between the largest and the smallest data is called the ***Rank***; and it is denoted by:

$$R = D\,M \text{ “major data"} - D.\,m. \text{"minor data"}$$

Now we will establish the ***Amplitude of the class interval,*** this is obtained by dividing the rank by the number of classes. Therefore, the amplitude of a class interval is the length of the interval.

$$Amplitude = \frac{R}{K}$$

Where: R is the Rank and **K** is the number of intervals

In our example the range is **R**= 100 – 45 = 55, and how **K**=6, we have that the amplitude has a value of:

$$Amplitude = \frac{55}{6} = 9.1666 \approx 9.17$$

Now we calculate the class intervals, start with the smallest data (45.0) and add the amplitude (9.17) to obtain the upper limit of the first interval (54.17), to this we add the amplitude to obtain the upper limit of the second interval (54.17 + 9.17 = 63.34), and so on until you have the total number of classes.

Intervals				**Frecuency** f_i
45.00	-	(45+9.17)=	54.17	3
54.17	-	(54.17+9.17)=	63.34	5
63.34	-	(63.34+9.17)=	72.51	10
72.51	-	(72.51+9.17)=	81.68	8

81.68	-	(81.68+9.17)= 90.85	5
90.85	-	(90.85+9.17)= 100.02	9
			40

Subsequently, a number that represents each of the generated intervals is chosen, this number is called class mark. The class mark is obtained as the semi-sum of the extremes of the interval, ie the limits of the interval or class and is denoted as xi:

The class mark of the first interval is obtained as: $\frac{45.0 + 54.17}{2} = 49.585$

The one in the second interval is: $\frac{54.7 + 63.34}{2} = 58.755$ and so on until the last class interval.

Intervals		**Frecuency** f_i	**Class Mark** $\bar{X}_i$	
45.00	- 54.17	3	(45+54.17)/2 =	*49.585*
54.17	- 63.34	5	(54.17+63.34)/2=	*58.755*
63.34	- 72.51	10	(63.34+72.51)/2=	*67.925*
72.51	- 81.68	8	(72.51+81.68)/2=	*77.095*
81.68	- 90.85	5	(81.68+90.85)/2=	*86.265*
90.85	- 100.02	9	(90.85+100.02)/2=	*95.435*

Activity 3. Analyze the example shown and write the procedure performed to solve it.

CLOSURE:

Activity 4. Team research (Co-evaluation)

Build a database on the variables of: age (years), height (cm.), Footwear measurement (Mex.) And gender (man / woman), of a group of students from your campus (each team must consider a different group to collect the information and with a minimum of 30 students), with which the following activities will be carried out: Remember that for the analysis of the information collected you must use the Information and Communication Technologies, since the work will be delivered in a electronic and printed for its exhibition and evaluation of the work done, (use of Word, Excel and PowerPoint application software).

1) Compilation of information of the data referring to the indicated variables.
2) Build a table with the information of the three study variables.
3) Organize the information for each of the variables separately so that using the formula Sturges determine the range and amplitude.
4) Make a frequency distribution table for each of the variables
5) Order and cleanliness in their jobs and have identified each activity performed by each member of the team
6) Exhibition of works.

Activity 5. Research work by team (Teams are made for the solution of the following problems that arise and at the end, present the constructed tables of each one.

1. The scores obtained by a group in a test have been:

15, 20, 15, 18, 22, 13, 13, 16, 15, 19, 18, 15, 16, 20, 16, 15, 18, 16, 14, 13.

a) Build the frequency distribution table and the class marks.

2. The number of stars of the hotels in a city is given by the following series:

3, 3, 4, 3, 4, 3, 1, 3, 4, 3, 3, 3, 2, 1, 3, 3, 3, 2, 3, 2, 2, 3, 3, 3, 2, 2, 2, 2, 2, 3, 2, 1, 1, 1, 2, 2, 4, 1.

a) Build the frequency distribution table and the class marks.

3. The 40 students in a class have obtained the following scores, over 50, in a Physics exam.

3, 15, 24, 28, 33, 35, 38, 42, 23, 38, 36, 34, 29, 25, 17, 7, 34, 36, 39, 44, 31, 26, 20, 11, 13, 22, 27, 47, 39, 37, 34, 32, 35, 28, 38, 41, 48, 15, 32, 13.

a) Build the frequency distribution table and the class marks.

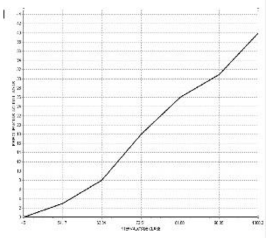
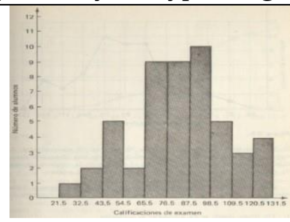
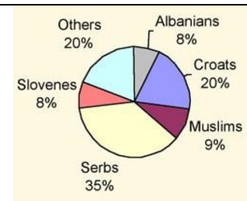
UNIT I

Didactic Sequence Num: __4_____ **DISTRIBUTION GRAPH**

Expected Learning: Represents, interprets and analyzes the information.	
Competences to develop:	
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M5 Analyze the relationships between two or more variables of a social or natural process to determine or estimate their behavior.
Generics:	2. He is sensitive to art and participates in the appreciation and interpretation of his expressions in different genres. 2.1 values art as a manifestation of the beauty and expression of ideas, sensations and emotions. 4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 4.5 Manages information and communication technologies to obtain information and express ideas. 7. Learn by initiative and self-interest throughout life. 7.2 Identify the activities that result in less and greater interest and difficulty, recognizing and controlling their reactions to challenges and obstacles. 8. Participate and collaborate effectively in diverse teams. 8.3 It assumes a constructive attitude, congruent with the knowledge and skills that it has within different work teams.
PRODUCTOS DE APRENDIZAJE: En equipo representar la información obtenida en la secuencia 3 mediante graficas de distribución de frecuencias.	

Opening	**Development**	**Closure**
Activity 1: Identification of graphs. (Self-evaluation) **Activity 2:** Elaboration of graphics. (Self-evaluation)	**Activity 3:** Summary (Heteroevaluation) **Activity 4:** Report of procedure analysis for table construction. (Heteroevaluation)	**Activity 5.** Resolving and graphing. (Heteroevaluation) **Activity 6.** Table of the solved exercises (Heteroevaluation)

OPENING: Activity 1. Individually identify the types of graphs presented.

<table>
<tr><td></td><td></td><td>
</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Activity 2. Graphing

Below is the data on the number of absences presented by students in a week of classes, it is necessary to perform the activity as a team of 2 to 4 people and make an Excel graphic corresponding to that information and send it by email to your teacher remember to make comments of what you see in the graph made.

Days	Monday. Tuesday. Wednesday, Thursay, Friday
Absences	15 5 10 5 20

DEVELOPMENT:

Activity 3. Perform the reading individually and make the corresponding summary in your notebook.

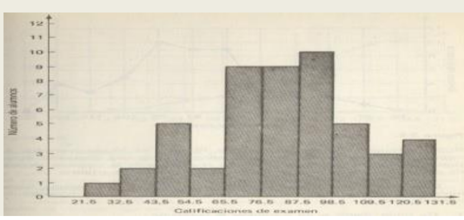
Graph of frequency distributions.

Graph: is an illustrated way to represent and summarize data.

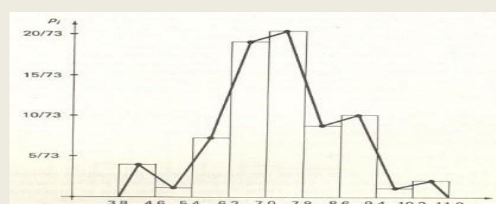
The majority of the graphical representations of statistical data, are existing relations between two variables that allow to project a descriptive curve, between these types of curves we have:

Histograms	The Ojiva
Frequency polygons.	Circular graphs

HISTOGRAMA



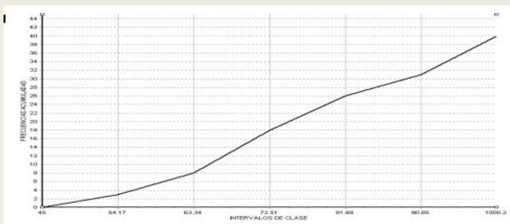
POLIGONO DE FRECUENCIAS



Some graphic representations of the collected data set or its frequencies can be exemplified in the following illustrations:

It is necessary to mention that currently, the drawing or drawing of a graph that represents the behavior of a study variable in a given population can be done by means of computer technology, either through the graphics tool or some other software specific for this purpose.

LA OJIVA

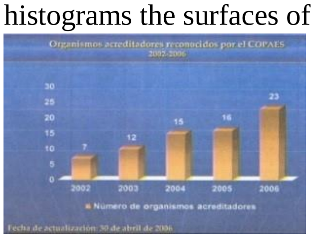


GRAFICAS CIRCULARES

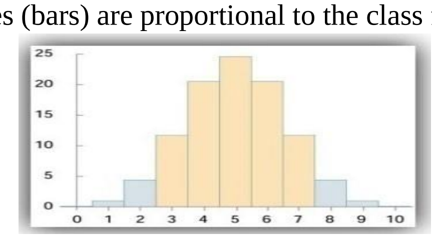
Histogram: It´s defined as the way to graphically represent a frequency distribution and that basically consists of a succession of rectangles that have their bases on the horizontal axis and length equal to the width of the class intervals, their heights are proportional to the class frequencies which are located on the vertical axis.

Although a histogram is very similar to bar charts, the following differences are conceptually noted:

- In a bar graph the heights of the same are related to the variable located on the vertical axis; while in the histograms the surfaces of the rectangles (bars) are proportional to the class frequencies.



Bar Graph



Histogram

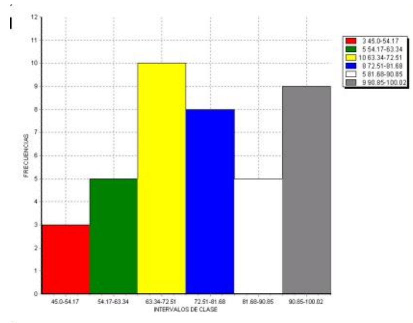
- In a bar chart these are graphed separately, that is, leaving spaces between each of them; while in the histograms the rectangles (bars) are represented consecutively.

For our case, we use the class intervals and their frequencies that were determined in the example developed above. These aspects are shown in the following table:

Intervals		**Frecuency** f_i
45.00 -	54.17	3
54.17 -	63.34	5
63.34 -	72.51	10
72.51 -	81.68	8
81.68 -	90.85	5
90.85 -	100.02	9
		40

On the abscissa axis "x" we establish the class intervals, and their frequencies are located on the axis of the ordinates "and". The width and height of each bar is established by the class interval and its respective frequency.

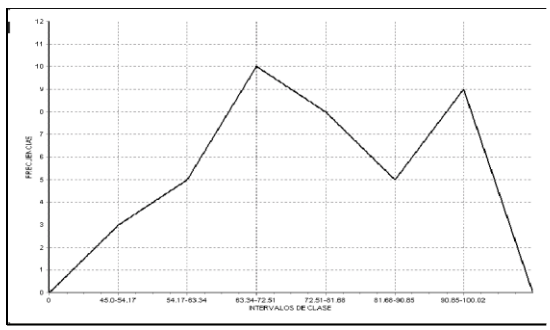
The behavior of the grades is given by the following histogram, where it is observed that a greater number of students obtained a grade between 63.34 to 72.51 and very few have a value of 45.0 to 54.17



Frequency polygon: It is another way to show the frequency distributions; said representation consists in that the class frequencies are indicated in the class marks or class midpoints, resulting in a series of points that are connected by means of straight lines.

In order to construct a polygon of frequencies, the class marks and the corresponding frequencies on the vertical axis are drawn successively on the horizontal axis, then the resulting points are joined by line segments, in addition two class marks with zero frequency are added, one at the beginning and the other at the end, which allow you to start and finish the graph on the horizontal axis.

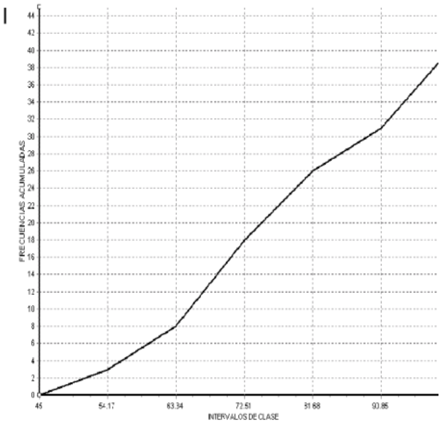
Intervals	Frecuency f_i	Class Mark x_i
45.00 - 54.17	3	49.585
54.17 - 63.34	5	58.755
63.34 - 72.51	10	67.925
72.51 - 81.68	8	77.095
81.68 - 90.85	5	86.265
90.85 - 100.02	9	95.435
	40	



Ojiva or polygon of accumulated frequencies: It´s a graph constructed with segments of straight lines, which join the points of the upper limits of each class interval and the accumulated frequencies or relative accumulated frequencies.

For its construction we start with a smaller number than the lower limit of the first class, which has zero frequency. Note that the warhead is non-decreasing by its nature.

Intervals			**Frecuency** f_i	**Accumulated Frecuency** (f_a)	**Relative accumulated Frecuency** (F_a)
45.00	-	54.17	3	3	3/40
54.17	-	63.34	5	8	8/40
63.34	-	72.51	10	18	18/40
72.51	-	81.68	8	26	26/40
81.68	-	90.85	5	31	31/40
90.85	-	100.02	9	40	40/40
			40		



Circular Graph: It is defined as the representation of data distributed in **percentage** form, that is, the circle is divided into sectors (slices of cake) that are equivalent in size to the corresponding percentage frequencies.

This graph is often used to represent economic situations, demographics, illness, etc.

The construction of this type of graphics has as its starting point the consideration that the total area corresponds to 360º equivalent to 100% of the circle; each portion or sector of area corresponds to a certain class of data, that is, it is a sector that represents a percentage equal to the ratio between the angle formed by the radii that limit it and the 360º of the circumference. Through the support of the conveyor the resulting portions are drawn.

Examples

1.- Based on the example developed above and considering the relative frequency obtained in each one of the study intervals, this allows to calculate the percentage contribution of said intervals, and these percentages in turn allow to construct their pie chart.

Intervals		**Relative Frecuency** (fr_i)	**% of the Relative Frecuency**	**Sexagesimal degrees by Interval**
45.00	- 54.17	3/40	7.5	27º
54.17	- 63.34	5/40	12.5	45º

63.34	- 72.51	10/40	25.0	90º
72.51	- 81.68	8/40	20.0	72º
81.68	- 90.85	5/40	12.5	45º
90.85	- 100.02	9/40	22.5	81º
		1	**100 %**	**360º**

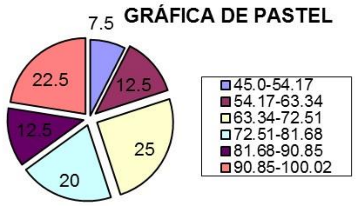
Considering that 360º is equivalent to 100% of the complete circle, we calculate that 7.5% is equivalent to a region of the circle with a value of 27º. Observe the following procedure, this same process we do in each of the intervals:

360º ______ 100%

x __________ 7.5%

$$x = \frac{(360^\circ)(7.5\%)}{100\%} = 27^\circ$$

With the obtained data we build the following frequency chart:



Graphic representation of the frequency of the final math grades of 40 students.

2.- The following table registers the surfaces of the five continents that make up the world.

CONTINENT		SURFACE (km^2)
AFRICA	30’ 224,000 AMERICA	42’ 198,760
ASIA		44’ 180,000
EUROPE		10’ 000,000
OCEANIA		08’ 970,000
TOTAL		135’ 572,760

Build the pie or pie chart.

a) Calculate the percentage contribution of each continent with respect to the world surface. In the case of the African continent this represents 22.3% and so on ...

$$Africa \frac{30224000}{135572760} \approx 22.3\ \%$$

b) We determine the opening of each region or portion that corresponds to each continent based on sexagesimal degrees.

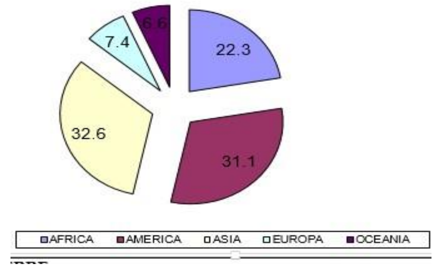
Considering that 360º is equivalent to 100% of the complete circle, we calculate that 22.3% eq a region of the circle with a value of 80. 28º, according to the following procedure. We do this in each of the continents:

360º ______ 100%

x __________ 22.3%

$$x = \frac{(360^\circ)(22.3\%)}{100\%} = 80.28^\circ$$

CONTINENT	SURFACE PERCENTAGE (%)	DEGREES SEXAGESIMALES
AFRICA	22.3	80.28º
AMERICA	31.1	111.96º
ASIA	32.6	117.36º
EUROPE	7.4	26.64º
OCEANIA	6.6	23.76º
TOTAL	100	360º



Activity 4. Make a report of the analysis of the procedure used for the construction of the frequency table, as well as the graphs.

CLOSURE: Activity 5. Resolving and graphing

The information presented corresponds to the number of absences of students during 50 days, based on the information presented below, resolves the missing columns and makes the graphs of: Histogram, Frequency Polygon, Ojiva and Pastel graphic.

Intervals	**Class Mark** X_i	**Frecuency** ***fi***	**Relative Frecuency fr**	**Accumulated Frecuency fa**	**Accumulated Relative Frecuency Fa**	**% of the Relative Frecuency**	**Sexagesimal degrees by interval**
30-40		**7**					
40-50		**5**					
50-60		**12**					
60-70		**8**					
70-80		**6**					
80-90		**8**					
90-100		**4**					
		50					

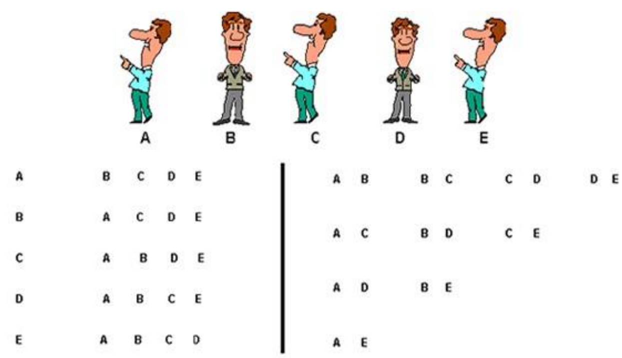
UNIT I

Didactic Sequence Num. ___5____	**DESCRIPTIVE MEASURES**
Expected Learning: Calculate the measures of central tendency.	
Competences to develop:	
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M5 Analyze the relationships between two or more variables of a social or natural process to determine or estimate their behavior. M8 Interpret tables, graphs, maps, diagrams and texts with mathematical and scientific symbols.
Genéric:	1. Know and value yourself and address problems and challenges taking into account the objectives pursued. 1.4 Critically analyze the factors that influence your decision making. 4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 5. Develops innovations and proposes solutions to problems based on established methods. 5.1 Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to the achievement of an objective. 8. Participate and collaborate effectively in diverse teams. 8.1 Proposes ways to solve a problem or develop a project in the team, defining a course of action with specific steps. 8.2 It contributes points of view with openness and considers those of other people in a reflexive way. 8.3 It assumes a constructive attitude, congruent with the knowledge and skills that it has within different work teams.

EXPECTED PRODUCTS: Argue that is a measure of central tendency through examples of such measures.		
Opening	**Development**	**Closure**
Activity 1: Observe and answer **Activity 2:** Questions **Activity 3:** Questions Co-evaluation	**Activity 4:** Summary **Activity 5:** Exercises **Activity 6:** Exercises Heteroevaluation	**Activity 7:** Fashion, medium and median exercises. **Co-evaluation**

OPENING:

Activity 1. Observe and answer



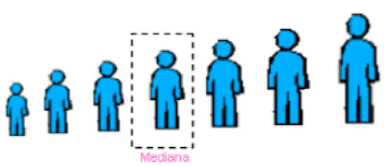
Look carefully at the following image and answer the questions below. Consider all the letters that are in the image and answer, how many letters exist of each type.

Gráfica la información de la tabla

Letter	Amount
A	
B	
C	
D	
E	
TOTAL	

Activity 2. Answer the questions that are asked regarding the image

As in the previous activity, he carefully observes the image presented and responds to the questions that are asked about it.



1. Why do you consider that one of the figures is highlighted?
2. How many figures exist on the right side of the one that is highlighted?
3. How many figures exist on the left side of the one that is highlighted?
4. What size will the highlighted figure have? (girl, medium, large)

Activity 3. As a group, answer the questions that are asked by pointing the answers on the board and reach a conclusion

1. What kind of phones are currently in greatest demand?

2. Can identify the characteristics of the mentioned phones:

3. Which of the phones do you consider most in demand?

DEVELOPMENT:

Activity 4. Read carefully the following information about the whole subject that is presented below, highlight what you consider most relevant and summarize it.

Descriptive measures

The resulting representations of the frequency distributions made it possible to discern the trends and patterns of the data. But what would happen if more descriptive measurements of a data set are needed? In this case, the numbers that are called descriptive measures would serve. These are all those measures that allow describing the population or a sample. In each case, then, what you get is:

For a population: are all those measures associated with a population, also called parameters.

For a sample: are all those measurements associated with a sample, also called statistics.

Descriptive measures are individual numbers that describe certain characteristics of populations or samples.

A set of data can be known numerically by means of some measures that describe it; for example, the mean, the standard deviation and others. In this way it is possible to compare several groups of data among themselves:

There are three types of descriptive measures:

I.- Measures of central tendency (or position): They are numerical values that locate, in some way, the center of a data set.

Among the different types of measures of central tendency we have:

Arithmetic mean or average **Medium** **Mode**

II.- Dispersion measures: These are numerical values that describe the dispersion or variability, which are found among the data.

Among the different types of dispersion measures we have:

• Range or amplitude **• Average deviation or average deviation** **• Variance**

• Standard deviation **• Coefficient of variation**

III.- Symmetry measurements: The curves presented by the observations in the data set can be symmetric or asymmetric. To measure this behavior we will have the bias.

Of the three types of measures we will only study those of central tendency and those of dispersion.

<u>I.- Measures of central tendency</u>

One of the most outstanding characteristics of data distribution is its tendency to accumulate towards the center of it. This characteristic is called central tendency.

The purpose of the central tendency measures is:

- Show where the average or typical person in the group is located.
- Serves as a method to compare or interpret any score in relation to the central or typical score.
- It serves as a method to compare the score obtained by the same person on two different occasions.
- It serves as a method to compare the average results obtained by two or more groups.

Arithmetic mean.	It´s the average used and that is usually called average. The arithmetic mean or average of a set of elements is defined as the sum of the values of these elements divided by the total number of them.

Expressed more intuitively, we can say that the mean (arithmetic) is the total amount of the variable distributed equally between each observation.

The **arithmetic mean** is determined by the equation:

Arithmetic mean	
Sample	**Population**
$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum x}{n}$	$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{\sum x}{N}$

Where:

$\bar{X}$ o μ = Average value of the data set. X = Value of each element

n ó N = Total number of elements of either a sample or a population

Examples 1) Calculate the arithmetic mean of the numbers 10, 11, 12, 12, 13

Data	**Formula**	**Substitution**	**Result**
n= 5	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	$\bar{X} = \frac{10+11+12+12+13}{5}$ $\bar{X} = \frac{58}{5}$	$\bar{X} = 11.6$

Therefore, the mean of the 5 numbers is 11.6. Note that the average results in a number that is between the range of elements; in this case, 11.6 is between 10.11, 12 and 13.

2) Obtain the measure of the price of oil registered in a month, if it was sold in the world market in 28, 31, 29, 27.26, dollars per barrel.

Data	**Formula**	**Substitution**	**Result**
n= 5	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	$\bar{X} = \frac{28+31+29+27+26}{5}$ $\bar{X} = \frac{141}{5}$	$\bar{X} = 28.2$

2) In an extraordinary examination of Mathematics and Physics, the grades obtained by a group of 13 students out of a maximum of 10 points were.

Course	Grades
Matemáticas	4,7,3,6,2,8,4,7,0,1,7,6,4.
Física	8,4,3,6,7,5,6,2,1,7,6,7,0.

Calculate the average of mathematics, physics and both subjects

Data	Formula	Substitution	Result
Mathematics 7 n= 13	4+ + + + + + + + + + + +7 3 0 17 64 $\bar{X} = 4.54$ / 13	$\bar{X} = \frac{59}{13}$	6 2 8 4
Physics n= 13	$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{n}$	$\bar{X} = \frac{8+ + + + + + + + + + + + +4\ 3\ 6\ 7\ 56\ 2\ 1\ 7\ 6\ 7\ 0}{13}$; $\bar{X} = 62\ n\ 13$	$\bar{X} = 4.76$
Both Subjescts n= 2	$\bar{X} = \frac{4.54+ 4.76}{2}$	$\bar{X} = \frac{9.3}{2}$	$\bar{X} = 4.65$

In a small maquila industry, the average salary of 8 workers is $ 875.00 biweekly.

4) Calculation of the monthly payroll amount.

Data	Formula	Substitution	Result
n = 8 Average per fortnight $875.00 Monthly average $\bar{X} = \$1750$	$\bar{x} = \frac{\sum_{i=1} x_i}{n}$ Clearance	$\sum X = (1750\ 8)\ ()$	The amount of the monthly payroll of the 8 workers is: $\sum X = 14000$

Activity 5: Solve in your notebook each of the following exercises remembering to perform the suggested steps:

1) Juan's grades in five subjects were: 85, 77.93, 76, 96. What is the arithmetic mean of his grades?

2) During a medical examination the reaction times of a person to certain stimuli were: 0.55, 0.42, 0.50, 0.48, 0.53, 0.50, 0.40, and 0.35 seconds respectively. Calculate the average reaction time of the person to the stimuli.

3) The monthly wages of 5 workers were: 2500, 3400, 1800, 5620, 4200. Calculate the arithmetic mean.

4) The price of a single room per day during the week in various hotels in the town was:
1342, 1148, 3170, 2650, 1320, 790, 2500, 2100, 1000, 1150
¿Cuál es el precio promedio de las habitaciones en los hoteles de la localidad?

5) The data presented below represent the ages of the patients admitted to a hospital on May 5, 1995: 3, 54, 63, 50, 85, 67, 53, 82, 83
Determine the average age (arithmetic mean) of the patients admitted to the hospital that day.

6) A track coach must decide. Which of the two riders must choose for the next 100 meter competition? The coach will base his decision on the results of the 5 races between the two riders, performed at 15-minute rest intervals. The following are the times recorded in the 5 races (in seconds).

	RACE				
ATHLETE	1	2	3	4	5
Daniel	12.5	12.1	11.7	16.9	12.9
Jaime	12.8	12.5	11.9	12.8	12.6

Considering the previous table of times, answer the following questions:

Which of the athletes should he choose? Would the decision change if the fourth race was not taken into account?

Median: It´s defined as the value that divides a set of previously ordered data (from least to greatest or vice versa) into two equal parts.

To calculate the median value, the following procedure is considered:

- If the set of elements ordered according to their magnitude is **ODD**, the median will be the intermediate value of said sequence.
- If the set of elements ordered according to their magnitude is **PAR**, the median will be the arithmetic mean of the two average elements.

Examples

1) Find the median of the following numbers: 2 4 1 3 5 6 3

First you have to sort the data from lowest to highest 1 2 3 3 4 5 6

By definition, the median of the numbers is the central value: 1 2 3 3 4 5 6

Therefore, the median of the data set is: 3

2) Find the median of the following numbers: 15, 13, 11, 14, 16, 10, 12, 18

First you have to sort the data from lowest to highest 10, 11, 12, 13, 14, 15, 16, 18

The median is the arithmetic mean of the numbers 10, 11, 12, 13, 14, 15, 16, 18

Therefore, the median of the data set is:

$$Median = \frac{13+14}{2} = 13\ 5.$$

Activity 6. In each of the exercises determine what is requested.

1) Find the median in the following data: 15, 21, 32, 59, 60, 60,61, 64, 71, 80.
2) Calculate the median of the results of a test: 69, 78, 90, 95, 80, 91, 51, 60, 64, 75, 70, 85
3) The data presented below represent the ages of patients admitted to a hospital on May 5, 1995. 3, 54, 63, 50, 85, 67, 53,82, 83 Calculate the average.
4) A supermarket chain compares prices for identical merchandise in all its grocery stores. Below are the prices of a kilogram of avocado that was sold in each store the previous week: \$5.00, \$7.00, \$ 6.40, \$7.20, \$ 8.10, \$ 7.50 Calculate the median price per kilogram of avocado.

Mode. It is defined as the value that is presented with the "highest frequency", that is, it is the "most common value" of a set of numerical elements.

In distributions not grouped in intervals the column of absolute frequencies is observed, and the value of the distribution to which corresponds the greater frequency will be the mode.

Among the characteristics of fashion, they emphasize that it may or may not exist, even if it exists it may not be unique.

If a given set of values presents a single mode it is called **UNIMODAL**; If it presents two modes, it is called **BIMODAL**, and if there are more than two modes, it is called **MULTIMODAL**.

Examples

1) Find the mode of: 5 12 9 5 8 7 1 We analyze the data set

Data	5	7	8	9	12

Repetitions	2	1	1	1	1

We observe that the most repeated data is 5.
Therefore, mode is 5 and the set is unimodal.

2) Given the following set of numbers: 4, 6,8,10,12,14,16,18,20 calculate the fashion We analyze the data set

Datos	4	6	8	10	12	16	18	20
Repeticiones	1	1	1	1	1	1	1	1

We observe that the elements of the set have equal frequency, reason why it is concluded that it does not have fashion that is to say it does not exist.

3) Given the following set of numbers 10,12,12,12,13,14,14,14,15,15,16,17,17,17, 19; Find your mode
Analyzing the data set

Datos	10	12	13	14	15	16	17	19
Repeticiones	1	3	1	3	2	1		1

We observe that the numbers that present most frequency are numbers 12, 14 and 17.
Therefore the mode is 12, 14 and 17 where the set is called multimodal.

4) In order to meet the wage demand of a group of 8 workers, their income is analyzed in pesos that are: 32, 40, 40, 45, 50, 55, 200, 300. Calculate the average, median and mode.

Average arithmetic — 32 + + + + + +40 40 45 50 55 200 + 300 762

$$\bar{X} = \frac{\sum fX}{N} \qquad \bar{X} = \frac{}{8} = \frac{}{8} = \$95\ 25.$$

Median The total set of numbers is 8 therefore the median is the mean of the numbers:

32, 40, 40, **45, 50**, 55, 200, 300 $\qquad Median = \frac{45+50}{2} = \frac{95}{2} == \$47\ 50.$

Mode

Data	32	40	45	50	55	200	300
Repetitions	1	2	1	1	1	1	1

The number that presents the most frequency is 40.
Therefore the fashion is 40.

Only 2 people have high incomes and the remaining 6 have salaries of $ 55.00 or less; The average presented a "deceptive" value, since the income of the workers is very dispersed with respect to the average value of income. The median of $ 47.50 and the $ 40.00 fashion are more representative.

80	52	92	75	82	96
70	90	69	83	94	67
61	96	88	63	78	83
85	75	81	73	97	109
100	85	95	88	98	78
98	76	100	58	108	89
88	64	81	70	105	64

CLOSURE:

Activity 7. Do the exercises individually in your notebook according to what is requested. 1) From the set of salary values shown in the following table, calculate: a) Arithmetic mean
b) the median of wages.
c) the modal salary.

2) Of the following grades obtained in a course determined:

a) arithmetic mean of the ratings
b) the median of the ratings.
c) the modal qualification

4	13	47	27	55	41	58	35	58	48
37	45	55	32	45	48	54	78	66	58
66	57	30	72	57	81	33	63	54	79
45	82	36	45	51	24	79	26	33	60
53	35	22	18	58	47	35	64	68	42

UNIT I

Didactic Sequence Num. ___6______	**DISPERSION MEASURES**
Expected Learning: Calculate dispersion measures	
Competences to develop:	
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M5 Analyze the relationships between two or more variables of a social or natural process to determine or estimate their behavior. M8 Interpret tables, graphs, maps, diagrams and texts with mathematical and scientific symbols.
Generic:	1. Know and value yourself and address problems and challenges taking into account the objectives pursued. 1.4 Critically analyze the factors that influence your decision making. 4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools. 4.1 Expresses ideas and concepts through linguistic representations, mathematical mathematics. 5. Develops innovations and proposes solutions to problems based on established methods. 5.1 Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to reaching a goal. 8. Participate and collaborate effectively in diverse teams. 8.1 Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps. 8.2 It contributes points of view with openness and considers those of other people in a reflexive way. 8.3 It assumes a constructive attitude, congruent with the knowledge and skills that it has within different work teams.

EXPECTED PRODUCT: Argue that is a measure of dispersion through examples and build quartiles from these data.		
Opening	**Development**	**Closure**
Activity 1: Exercise Sel-evaluation	**Activity 2.** Summary **Activity 3.** Exercises **Activity 4.** Exercises Peer evaluation activities	**Activity 5.** Closing exercise

OPENING:

Activity 1. Do the following exercise that is presented below, consider the table for it.

Next, a series of data is presented.

Disorderly table

3	3	4
6	10	6
8	9	7

Sorted table

What is the smallest data?

What is the biggest data?

What is the difference between the data (larger and smaller)?

What is the data that is repeated most frequently (fashion)?

What is the data found in the center (median)?

What is the average data (average)?

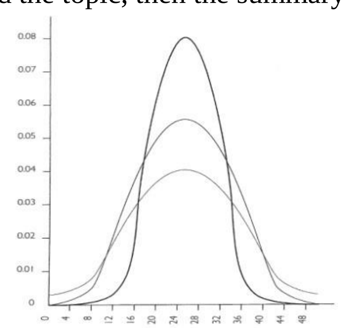
Compare the answers with your classmates and perform a self-assessment of the activity.

DEVELOPMENT:

Activity 2. For the development of this activity it is necessary that you first read the topic, then the summary.

Measures of dispersion

Using the arithmetic mean, the median and the mode as measures of central tendency is not enough, since they only describe the center of the distribution, but not of dispersion as can be seen in the figure.



It is observed in this graphic representation, that in these curves the mean, the median and the mode have a value equal to 25; but in no way are their distributions the same, as can be seen in their own distributions. Here we find a new property to measure: the variability or dispersion.

Variability is a very important feature of a data set. For example:

 If a drug is manufactured where the variation in the dose is very large, this implies that a high percentage of doses will be discarded because it is a risk factor for health.

 Or if a group of financial analysts detect widely dispersed earnings (ranging from very small securities to extremely large values), these will be an indicator of the risk that exists for shareholders and creditors. These cases are more striking than those where profits remain relatively stable.

But, ***why is variability important ?:***

 Because it provides additional information that allows judging the reliability of our measure of central tendency, since, if the data are very dispersed, then the variability that exists between them will be very large;

however, without a narrow set, the variability will be very small. And the average will have a greater accuracy when taken as a measure of central tendency.

 Because it allows comparisons between different samples or populations.

 Because it allows to distinguish sets of data that present wide or narrow variations in different samples as well as in different populations.

The most common dispersion measures are:

Range or amplitude	**Mean deviation or average deviation**	**Variance**
Standard deviation	**Coefficient of variation**	**Other measures of dispersion (quartiles, deciles and percentiles)**

Range or amplitude. It´s defined as the difference between the largest data (DM) and the smallest data (dm) of the distribution.

RANK	
Sample	**Population**
$R = D_M - d_m$	$R_p = D_M - d_m$

Unfortunately this measure is not very satisfactory since it can be that one data set has the same rank as another and they are very different from each other.

Examples

1) Find the range of the following data sets:
   a) 2 2 2 2 2 3 4 4 4 4 4 4 4 4 4 4 5 6 10

Mayor data is: 10 and the minor data is 2 $Rank = 10 - 2 = 8$

2) Find the range for the following data sets:
   a) 16, 10, 11, 7, 19,14, 22, 9

Sorting in ascending order the elements of the set, we have:7, 9, 10, 11, 14, 16, 19, 22

Mayor data is: 22 y el dato menor es 7 $Rank = 22 - 7 = 15$

Mean deviation or average deviation

Average deviation. It is defined as the arithmetic mean of the absolute values of the deviations of the variables with respect to the arithmetic mean.

$$Average\ deviation = DM = \frac{\sum_{i=1}^{N} \left| X_i - \overline{X} \right|}{N} = \frac{\sum \left| X - \overline{X} \right|}{N} =$$

Where:

$\left| X - \overline{X} \right| =$ Absolute value of the deviations of the different numerical elements of its arithmetic average.

N = Total number of elements of the set.

The mean or average deviation of deviation has disadvantages in terms of the measure of central tendency that is used for its calculation:
The most serious problem occurs when the signs of deviations are not taken into account (because they are absolute values), which does not show if the deviation is above or below the arithmetic mean. If the sign is respected, the sum of the deviations with respect to the mean is zero and approaches zero when the deviations are with respect to the median; therefore, **the average or average deviation of deviation** should be called "**absolute average deviation".**

Examples:
1) Find the average deviation of the following set of numerical elements: 5, 8, 11, 13, 17, 21, 24 First you have to calculate the arithmetic mean of the set of numerical elements:

$$\bar{X} = \frac{\sum X}{N} \quad \bar{X} = \frac{5+8+11+13+17+21+24}{7} \quad \bar{X} = \frac{99}{7} = 14.14$$

Now you have to calculate $DM = \frac{\sum |X - \bar{X}|}{N}$

$$DM = \frac{|5-14.14| + |8-14.14| + |11-14.14| + |13-14.14| + |17-14.14| + |21-14.14| + |24-14.14|}{7}$$

$$DM = \frac{9.14 + 6.14 + 3.14 + 1.14 + 2.86 + 6.86 + 9.86}{7} \quad DM = \frac{39.14}{7} \quad DM = 5.59 \approx 5.6$$

Therefore, the average deviation of the set of numerical elements is 5.6

1) Find the average deviation of the next set of numerical elements
1 5 7 7 8 9 9 10 17
First we must calculate the arithmetic mean of the set of numerical elements:

$$\bar{X} = \frac{\sum X}{N} \quad \bar{X} = \frac{1+5+7+7+8+9+9+10+17}{9} \quad \bar{X} = \frac{73}{9} = 8.11$$

Now you have to calculate $DM = \frac{\sum |X - \bar{X}|}{N}$

$$DM = \frac{|1-8.11| + |5-8.11| + |7-8.11| + |7-8.11| + |8-8.11| + |9-8.11| + |9-8.11| + |10-8.11| + |17-8.11|}{9}$$

$$DM = \frac{7.11 + 3.11 + 1.11 + 1.11 + 0.11 + 0.89 + 0.89 + 1.89 + 8.89}{9} \quad DM = \frac{25.11}{9} \quad DM = 2.79 \approx 2.8$$

Therefore the average deviation of the set of numerical elements is 2.8

Variance and Standard Deviation

The notions of variance and standard deviation are used to quantify the variability of a sample by measuring its dispersion around the mean.

Variance. The variance (S^2) of a set of elements $X_1, X_2, X_3, \ldots, X_N$ it is defined as the average arithmetic of the squares of deviations from the arithmetic mean.

Variance

Sample	Population
$s^2 = \dfrac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{n-1}$	$\sigma^2 = \dfrac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{N}$

Standard deviation.	The standard deviation (S) of a set of elements $X_1, X_2, X_3, \ldots, X_N$ it is defined as the square root of the mean square of the deviations to the mean, that is, it is the square root of the variance.

Standar Deviation

Sample	Population
$s = \sqrt{\dfrac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{n-1}}$	$\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{N}}$

Some important properties of the standard deviation, which are what we will use the most for its easy interpretation are:

• The value of the standard deviation is always positive or equal to zero: $s \geq 0$.

• When it is equal to zero, there is no dispersion. This could only happen if all the data are equal, which would be related to the analysis of a variable that is constant, or something similar: $s = 0$. This is the minimum variation of any data set.

• The greater the number of the variance or the standard deviation, the greater the value of the quadratic deviations with respect to the mean. For example, if a sample has $s = 5$ and another sample has $s2 = 1$, this implies that there is greater dispersion in the first sample than in the second sample. That is, the deviations with respect to the average ones are greater than the deviations of the sample two with respect to their respective average.

Example

1) Find the variance and the standard or standard deviation for the following set of numerical data: 8, 15, 11, 5, 10, 12, 8 and 13

Step 1 Look for	Step 2 Look for	Step 3 Look for each	Step 4 Look for	Step 5 Sample Variation
$\sum X$	$\bar{X} = \sum X$	$\left(X - \bar{X}\right)$	$\sum\left(X - \bar{X}\right)^2$	$s^2 = \sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

n $n-1$

8 —8 – 10.25 = - $\frac{82}{8}$ 2.25 $(-2.25)^2$ =5.06 15 $X=$ 15 – 10.25
= 4.75 $(4.75)^2$ =22.56

11 11 – 10.25 = 0.75 $(0.75)^2$ =0.56

5		5 – 10.25 = -5.25	$(-5.25)^2$ =27.56
10	$\bar{X}=10.25$	10 – 10.25 = -0.25	$(-0.25)^2$ =0.06
12		12 – 10.25 = 1.75	$(1.75)^2$ =3.06
8		8 – 10.25 = -2.25	$(-2.25)^2$ =5.06
13		13 – 10.25 = 2.75	$(2.75)^2$ =7.56

$$s^2 = \frac{71.67}{8-1}$$

$$s^2 = \frac{71.67}{7}$$

$$s^2 = 10.23$$

$\sum X = 82$ $\sum (X - \bar{X})^2 = 71.67$

Therefore, the variance of the data set is 10.23.
To calculate the standard deviation we just have to get the root of the variance.

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}} \qquad s = \sqrt{10.23} = 3.19$$

The standard deviation is 3.19

Coefficient of variation.	It´s a measure of relative dispersion, which is obtained by dividing the standard deviation of a set of data by its arithmetic mean.

Coefficient of variation

$$C.V. = \frac{s}{\bar{x}}$$

The coefficient of variation has the following characteristics:

- The coefficient of variation is a dimensionless measure (that is, without dimension).
- It is the most appropriate measure to compare the variability of two data sets.

By multiplying the value of the coefficient of variation by 100, we obtain the C.V. in percentage form.

Example

1)The arithmetic mean of the wages in pesos of an automotive company is
14 600 and its standard deviation is 147; In a maquiladora company in which they manufactured carburetors for automobiles, the arithmetic mean of wages is 5700 and their standard deviation is 38. Calculate the two series and decide which of the two has greater wage variation.

Automotive industry

Data: $\bar{X} = 14600$ $s = 147$ substituting the data $CV = \frac{s}{\bar{X}}$ Then $CV = \frac{147}{14600} = 0.010 = 1\%$

Maquiladora industry Data: $\bar{X} = 5700$ $s = 38$ substituting the data $CV = \frac{s}{\bar{X}}$

We have that: $CV = \frac{38}{5700} = 0.006 = 0.6\%$

As you can see, the automotive industry has greater variation in salaries.

Activity 3. Perform the exercises individually in your notebook. **Evaluate a partner**. Remember to make constructive comments in case of having incorrect exercises and make the modification

Find the **range or amplitude** for the following number sets

1) 11, 5, 6, 2, 14, 9, 17,4
2) 16,10,11,7,19,14,22,9
3) 22,13,12,13,12,12,7,13
4) 12, 6, 11, 11, 12, 11, 12, 19, 13, 5
5) 85, 92, 73, 69, 89, 78, 69, 96, 90

Find the **average deviation** for the following number sets:

1) 11, 5, 6, 2, 14, 9, 17,4
2) 16,10,11,7,19,14,22,9
3) 22,13,12,13,12,12,7,13
4) 12, 6, 11, 11, 12, 11, 12, 19, 13, 5
5) 85, 92, 73, 69, 89, 78, 69, 96, 90

Find the **variance, the standard deviation and the coefficient of variation** given the following samples

1) 2, 4, 7, 8, 9.
2) 7, 6,10,7,5,9,3,7,5,13
3) The weights of 5 students are as follows: 60kg, 75kg, 55kg, 58 kg, 80kg.

4) 15 students from CECyTE, N.L., Randomly selected, were asked to mention the number of hours they slept last night. The results were 5,6,6,8,7,7,9,5,4,8,11,6,7,8,7.

5) The following data represents the monthly income in thousands of pesos of a group of 20 employees of a certain city.

2.0	1.6	2.3	2.8	2.8	3.1	1.1	2.1	2.7	1.7
1.9	1.4	1.3	1.7	1.8	1.1	1.9	2.0	2.3	2.0

What can you say about the variability of the income of this group of people?

Other measures of dispersion

The standard deviation is the most widely used dispersion measure. However, there are other ways to describe the variation or dispersion of a data set. One method is to determine the location of the values that divide a set of observations into equal parts. These measures include **quartiles, deciles and percentiles.**

The quartiles: divide a set of observations into four equal parts. To exemplify we can think of a set of values ordered from lowest to highest, once ordered the intermediate value of the data set is the median, which is a measure of location, since it points to the data center. Similarly, quartiles divide a set of observations into four equal parts.

The first quartile represented by Q1 is the value below which 25% of the observations are presented and the third quartile, represented by Q3, is the value below which represents 75% of the observations. It is logical that Q2 is the median. Q1 can be considered as the median of the lower half of the data and Q3 as the median of the upper part of the data.

The deciles divide a set of observations into 10 equal parts.

The percentiles divide the data set into 100 equal parts. Therefore, if a general average in the university is in the eighth decile, it can be concluded that 80% of the students had a general average lower than that average and that 20% will only be the average higher.

Quartiles, deciles and percentiles

To formalize the calculation process, suppose that Lp represents the location of a certain percentile that is sought.

So, if you find the thirteenth percentile, use L33, and look for the median, the 50th percentile, then L50. The number of observations is n, so if you want to locate the median, your position is in (n + 1) / 2, or you could write this expression as (n + 1) (P / 100), in which P represents the percentile that is sought.

Location of a percentile $LLLL = (nn + 1)\frac{PP}{100}$

Example: The commissions that he won last month, a sample of 15 stock brokers, are presented in an office in Monterrey. This investment company has offices throughout the country.

$2,300	$1,758	$1,721	$1,637	$2,097	$2,205	$1,787	$2,287
$1,940	$2,311	$2,054	$2,406	$ 2,047	$1,471	$1,460	

Find the median, the first and the third quartile of the generated commissions.

1. Sort the data from the smallest commission to the largest.

$1,460	$1,471	$1,637	$1,721	$1,758	$1,787	$1,940	$2,047
$2,054	$2,097	$2,205	$2,287	$2,300	$2,311	$2,406	

The median value is the observations found in the center. The central value or L50, is located in (n + 1) (50/100), where n represents the number of observations. In this case position 8 determined by (15 + 1) (50/100). The eighth largest commission is $ 2,047, so it's the median and half the brokers get commissions greater than $ 2,047 and the lesser half less than $ 2,047.

Recordemos la definición de cuartil. Los cuartiles dividen a un conjunto de observaciones en cuatro partes iguales. Por consiguiente, 25% de las observaciones serán menores que el primer cuartil, 75% de las observaciones serán menores del tercer cuartil.

To locate the first quartile we have to: n=15 y P=25.

$$LLLL = (nn+1)\frac{PP}{100} \quad LL_{25} = (15+1)\frac{25}{100} = (16)(.25) = 4$$

To locate the fourth quartile we have to: n=15 y P=75.

$$LLLL = (nn+1)\frac{PP}{100} \quad LL_{75} = (15+1)\frac{75}{100} = (16)(.75) = 12$$

Por lo tanto, los valores del primer y tercer cuartil se localizan en las posiciones 4 y 12, los cuales el cuarto valor para el primer cuartil le correspondería $1,721 y para el tercer cuartil sería $2,205.

On the other hand, if there were 20 observations in the sample, that is, n = 20 and you would like to locate the first quartile?

It would be : $LLLL = (nn+1)\frac{PP}{100} \quad LL_{25} = (20+1)\frac{25}{100} = (21)(.25) = 5.25$

It would locate the fifth value in the ordered series and then move a distance of 0.25 between the fifth and sixth values and inform it as the first quartile. As in the case of the median, the quartile does not need to be one of the exact values of the data set.

For example: suppose that a data set contains the six values 91, 75, 61, 101, 43, and 104. The first quartile is located in $LLLL = (nn+1)\frac{PP}{100} \quad LL_{25} = (6+1)\frac{25}{100} = (7)(.25) = 1.75$

The location indicates that the first quartile is located in the first and second values, which represents 0.75 of the distance between the first and second values.

Activity 4. Do the following exercises described below

1. Determine the median and the values corresponding to the first and third quartiles, with the values presented.

46	47	59	55	49	53	54	55	49	54	55

2. Determine the median and the values corresponding to the first and third quartiles, with the values presented.

5.24	9.61	6.67	10.39	7.59	12.71	8.03	13.59	8.81	9.45
6.02	10.37	7.30	11.86	12.22	7.99	13.07	8.35	13.89	15.42

3.Below are the different times that 30 customers take to pay their water bill.

13 41 41 45 20 27 31 34 51 34 53 35 56 36 38
41 13 13 47 26 47 47 34 50 51 34 54 35
62 82

a) Determine el primer y tercer cuartiles
b) Determine el segundo y el octavo decil
c) Determine el 67º percentil

CLOSURE: Activity 5. Solve the following problem individually

Every Saturday the sales staff is asked to make a report of the number of calls they make to their customers, which is presented in the following table.

38	48	51	53	55	62	62	66	69	65

40 48 51 54 56 59 62 67 69 78 41 50 52
55 57 59 63 66 71 79

45	50	52	55	56	59	64	67	77	79

a) Determine the median number of calls

b) Determine the first and third quartiles

c) Determine the first and ninth deciles

d) Determine the 33rd percentile

UNIT II

Didactic Sequence Num. **7**	**MEASURES OF FORM**

Expected learning: Calculate the measures of form and interpret the measures of central tendency from the analysis of the statistical graph, as well as its variability and representation of the contextual situation.

Competences to develop:	
Discipline	M2 Formulate and resolve math problems , apply differents approache. M3. Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Arguments the solution obtained of a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M6. It quantify, represents and contrasts experimentally or mathematically the magnitudes of space and the physical properties of the objects that encircle. M7. Choose a deterministic or a random approach to the study of a process or phenomenon, and argue its propriety.
Generics:	1.1 Confront the difficulties that present and is conscious of their values, fortitude and weakness. 1.6 Manage available resources taking account the restriction for the achievement in the goals. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations.. 4.5 Manages information and communication technologies to obtain information and express ideas. 5.3 Identify the systems and rules or core principles that underlie a series of phenomena. 7.2 Identfy the activities that result in minor and greatest interest and difficulty, and recognise challenges and obstacles.

EXPECTED PRODUCT: From the obtaining the measures of central tendency in the exercises, classify the data in some type of distribution.

Opening	**Development**	**Closure**
Activity 1. Exercise of opening (self appraisal) **Activity 2.** Graphic histogram (self appraisal)	**Activity 3.** Conceptual map (Heteroevaluation) **Activity 4.** Exercises of average, median and mode. (Coevaluation) **Activity 5.** Exercises of moments (Coevaluation)	**Activity 6.** Questions (Heteroevaluation) **Activity 7.** Exercises of closing (Heteroevaluation)

Opening

Activity 1. Analyze individually the next problem, and resolve in your notebook: In order to care for the wage claim of group of 8 workers, your income in pesos is: 32, 40, 40, 45, 50, 55, 200, 300.

a) Obtain the average, median and mode of the proportional data.

b) Compare the value obtained the average regarding to the median and mode. ¿How is largest, lower or identical?

Activity 2. Below present a table of information for make a histogram graph, after answer the questions.

# of siblings	Frequency
0-1	15
2-3	10
3-4	4
5-6	2

In a classroom applied differents polls at the students and some results are:

a) Observe the graph applied and answer ¿where is the most data?

b) ¿the graph is symmetric?

¿Why?

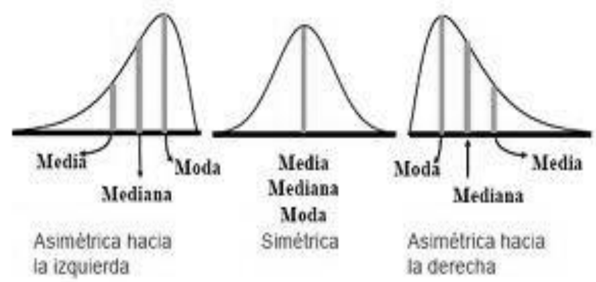
Finished the activity, make in way of brainstorming the results obtained in each of the activities.

DEVELOPMENT:

Activity 3. Read individually the topic and make a conceptual map.

Form measurements allow check if the frequency distribution has special characteristics like, symmetry, asymmetry, level of data concentration and pointing level that classify in a particular type of distribution.

Form measurements: Are statistical indicators that allow to identify, if a frequency distribution presents uniformity, can be classified into two large groups: bias measures and aiming measures.



Symmetry measurements: When the value of the variable that equidistant of a central value is the same frequencies. In this fact is verified:

$\bar{x} = Me = Mo$ where: $\bar{x}$ is the average, Me is the median and Mo is the mode.

Bias or asymmetry measurements: Informs without the extreme of the curves (tails) associated at the datum are more long to some of the sides.

The asymmetries can has bias:

- Positive or right: it has the frequencies more top on the left of the average and the more small on the right (tails)

$Mo < Me < \bar{x}$ ó $Mo = Me < \bar{x}$

- Negative or left: it has the frequencies more top on the right and more small on the lefth (tails)

$\bar{x} < Me < Mo$ ó $\bar{x} < Me = Mo$

Examples:

1) Throughout a week of the month april 2010 the contribution of dollar regarding to the peso were:

12.33, 12.26, 12.24, 12.22, 12.24, 12.19, 12.24

a) Calculate the average	b) Obtain the median	c) The mode

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\bar{x} = \frac{12.33+12.26+12.24+12.24+12.24+12.22+12.19}{7}$ $\bar{x} = \frac{85.72}{7} = 12.24$	12.19 12.22 **12.24** **12.24** 12.24 12.26 12.33 Me = 12.24	Mo = 12.24

If we compare the three measurements of central tendency can say that are equal for the datum distribution is symmetric, and conclude the dollar had a normal behavior.

2) To write an article about the types of printers available, the following dollar prices of the available models.575 259 550 340 475 520 550 398

a) Calculate the average	b) Obtain the median	c) The mode
$\bar{x} = \frac{+340+475+520+550+398}{8}$ $\bar{x} = \frac{567}{8} = 458.375$	259 340 398 **475** **520** 550 550 575 $Me = \frac{475+520}{2} = 497.5$	Mo = 550

If we compare the three measurements of central tendency, can say that $\bar{x} < Me < Mo$ so the distribution is negative asymmetry for left.

3) In order to the wage demand of a group of 8 workers, is necessary analyze your income in pesos that are: 32, 40, 40, 45, 50, 55, 200, 300

a) Calculate the average	b) Obtain the median	c) The mode:
$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\bar{x} = \frac{32+40+40+45+50+55+200+300}{8} = \frac{762}{8} = \$95.25$	32, 40, 40, **45, 50**, 55, 200, 300 $Me = \frac{45+50}{2} = \frac{95}{2} = \$47.50$	Mo = 40

If we compare the three measurements of central tendency, can say that $Mo < Me < \bar{x}$ so the distribution is positive asymmetry for right.

Activity 4. Individual calculate the average, median and mode, and compare the results obtained and indicated, if the results has a symmetry or asymmetry.

1) A psychologist wrote a computer program to simulate the way a person fills a test IQ standard, to test the program, he entered into the computer 15 different forms of an IQ test whose known and calculated coefficient in each form.

134	144	138	146	148
143	137	135	153	146
136	144	138	147	146

2) They were selected 13 students said the number of hours that sleep the results were 5, 6, 6, 7, 7, 9, 5,4, 11, 6, 7, 8, 7.

3) In the next table´s concentratethe qualifications obtained in the final statistics exam for a group of 45 students:

Qualifications	2	3	4	5	6	7	8	9	10
Students	3	1	2	4	10	13	5	6	1

The values $12, $15, $13, $17, $15, $18, $15, $13 y $17.

Fisher coefficient: Fisher´s asymmetry coefficient is defined for:

$$S_f = \frac{\frac{1}{n}\sum_{i=1}^{k}(x_i - \bar{x})^3 n_i}{S^3}$$

Where:

S_f Fisher´s of asymmetry coefficient. S Is the typical deviation. n number of observations.

The interpretation is clear and is the following:

If $S_f = 0$, so the distribution is symmetry.

If $S_f > 0$, so the distribution is asymmetry for right.

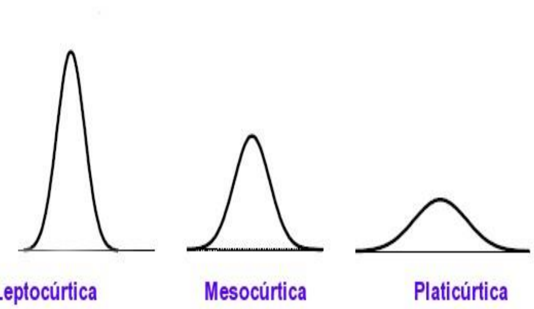
If $S_f < 0$, so the distribution is asymmetry for left.

Measures of aiming or Kurtosis: aThe Word curto comes from the latin ´curtus´, which means short or diminished. The word ´curtois´ is used to refer measures of form the distributions distinguish between:

Leptocurtic: Distributions more pointed that normal.

Mesocortic: Distributions with normal aiming.

Platicurtic: Distributions less pointed that normal. Fisher´s pointing coefficient is used to measure the greater or minor pointing, this is not the only coefficient, but it is the most used and is defined as:



$$A_f = \frac{\frac{1}{n}\sum_{i=1}^{k}(x_i - \bar{x})^4 n_i}{S^4} - 3$$

Where: A_f Fisher´s aiming coefficient.

S Is the standard deviation

n Is the number of observations.

The interpretation is clear and is as follows:

If $A_f = 0$, so teh distribution is mesocurtic.	If $A_f > 0$, so the distribution is leptocurtic.	If $A_f < 0$, so the distribution is la platicurtic.

Example:

1) The hemoglobin in 100 ml of a group of patients is shown in the following table.

	x_i	f_i	$x_i f_i$
8-10	9	4	36
10-12	11	10	110
12-14	13	25	325
14-16	15	12	180
16-18	17	8	136

a) Calculate the average and standard deviation $\bar{x} = \frac{\sum f_i x_i}{N}$

$$\bar{x} = \frac{36+110+325+180+136}{59} = \frac{787}{59} = 13.34$$

Find $(x - \bar{x})$	Find $\sum (x - \bar{x})^2$	$\sum f_i (x - \bar{x})^2$	Variance $s = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})}{}$
9-13.34=-4.34	$(-4.34)^2$ =18.83	(4)(18.83)=75.32	
11-13.34=-2.34	$(-2.34)^2$ =5.47	(10)(5.47)=54.7	$s^2 = \frac{273.03}{59}$
13-13.34=-0.34	$(-0.34)^2$ =0-1156	(25)(0.1156)=2.89	
15-13.34=1.66	$(1.66)^2$ =2.75	(12)(2.75)=33	
17-13.34=3.66	$(3.66)^2$ =13.39	(8)(13.39)=107.12	$s^2 = 4.62$ $s = 2.15$
		$\sum f_i (x - \bar{x})^2 = 273.03$	

b) Calculate the Fisher´s asymmetry coefficient. $S_f = \frac{\left(\frac{1}{n}\right) \sum_{i=1}^{k} (x_i - \bar{x})^3 n_i}{s^3}$

Find $(x - \bar{x})$	$\sum (x - \bar{x})^3 n_i$	
9-13.34=-4.34	$(-4.34)^3(4)$=-326.98	$S_f = \frac{\left(\frac{1}{59}\right)(8.97)-}{(2.15)^3} = \frac{-0.1520}{9.93} = -0.015$
11-13.34=-2.34	$(-2.34)^3(10)$=-128.12	
13-13.34=-0.34	$(-0.34)^3$ (25)=-0.9826	
15-13.34=1.66	$(1.66)^3$ (12)=54.89	
17-13.34=3.66	$(3.66)^3$ (8)=392.22	
	$\sum (x - \bar{x})^3 n_i = -8.97$	As $S_f < 0$, so the deviation the distribution is asymmetry of left.

c) Calculate the aiming coefficient. $A_f = \dfrac{\left(\frac{1}{n}\right)\sum_{i=1}^{k}(x_i - \bar{x})^4 n_i}{s^4} - 3$

Find $(x - \bar{x})$	$\sum(x - \bar{x})^4 n_i$	
9-13.34=-4.34	$(-4.34)^4(4)=1419.11$	$A_f = \dfrac{\left(\frac{1}{59}\right)(3245.904)}{(2.15)^4} - 3$
11-13.34=-2.34	$(-2.34)^4(10)=299.82$	$A_f = \dfrac{55.01}{21.36} - 3 = 2.57 - 3 = -0.42$
13-13.34=-0.34	$(-0.34)^4(25)=0.3340$	If $A_f < 0$, so the distribution is platicurtic that is more flatten that a curve normal.
15-13.34=1.66	$(1.66)^4(12)=91.11$	
17-13.34=3.66	$(3.66)^4(8)=1435.53$	
	$\sum(x - \bar{x})^4 n_i = 3245.904$	

2) There is data about the start time of an engine in seconds: 1.75; 1.92; 2.62; 2.35; 3.09; 3.15; 2.53; 1.91. Calculate

a) the average $\bar{x} = \dfrac{\sum f x_i}{N} = \dfrac{1.75+1.92+2.62+2.35+3.09+3.15+2.53+1.91}{8} = \dfrac{19.32}{8} = 2.415\ seg.$

b) Standard deviation

1.75-2.415=-0.665	$(-0.665)^2=0.4422$	$s^2 = \dfrac{1.9932}{8}$
1.92-2.415=-0.495	$(-0.495)^2=0.2450$	
2.62-2.415=0.205	$(0.205)^2=0.04202$	
2.35-2.415=-0.65	$(-0.65)^2=0.4225$	$s^2 = 0.2415$ $s = 0.4991$
3.09-2.415=0.675	$(0.675)^2=0.4556$	
3.15-2.415=0.735	$(0.735)^2=0.5402$	
2.53-2.415=0.115	$(0.115)^2=0.0132$	
1.91-2.415=-0.505	$(0.505)^2=0.2550$	
	$\sum(x - \bar{x})^2 = 1.9932$	

c) Calculate the Fisher´s asymmetry coefficient. $S_f = \dfrac{\left(\frac{1}{n}\right)\sum_{i=1}^{k}(x_i - \bar{x})^3 n_i}{s^3}$

$(x - \bar{x})$	$\sum(x - \bar{x})^3$	
1.75-2.415=-0.665	$(-0.665)^3=-0.2940$	$\left(\frac{1}{8}\right)(0.15352)$
1.92-2.415=-0.495	$(-0.495)^3=-0.1212$	

2.62-2.415=0.205	$(0.205)^3=0.0086$	$S_f = \frac{0.1919}{(0.4991)^3} = \frac{0.1919}{0.1243} = 1.54$
2.35-2.415=-0.65	$(-0.65)^3=-0.2746$	
3.09-2.415=0.675	$(0.675)^3=0.3075$	
3.15-2.415=0.735	$(0.735)^3=0.3970$	
2.53-2.415=0.115	$(0.115)^3=0.00152$	
1.91-2.415=-0.505	$(0.505)^3=0.1287$	
	$\sum(x-\bar{x})^3 = 0.15352$	As $S_f > 0$, so the distribution is asymmetry of right.

d) Calculate the aiming coefficient.

$$A_f = \frac{\left(\frac{1}{n}\right)\sum_{i=1}^{k}(x_i - \bar{x})^4 n_i}{s^4} - 3$$

$(x-\bar{x})$	$\sum(x-\bar{x})^4$	
1.75-2.415=-0.665	$(-0.665)^4=0.1955$	$A_f = \frac{\left(\frac{1}{8}\right)(1.00017)}{(.4991)^4} - 3$
1.92-2.415=-0.495	$(-0.495)^4=0.0600$	$A_f = \frac{0.1250}{0.06205} - 3 = 2.01 - 3$
2.62-2.415=0.205	$(0.205)^4=0.0017$	$= -0.98$
2.35-2.415=-0.65	$(-0.65)^4=0.1785$	If $A_f < 0$, so the distribution is that more flatten that a normal curve.
3.09-2.415=0.675	$(0.675)^4=0.2075$	
3.15-2.415=0.735	$(0.735)^4=0.2918$	
2.53-2.415=0.115	$(0.115)^4=0.00017$	
1.91-2.415=-0.505	$(0.505)^4=0.0650$	
	$\sum(x-\bar{x})^4 = 1.00017$	

Activity 5. Individual calculate Fisher´s asymmetry and aiming coefficient with the information below.

1) The data are: 2, 3, 9,2

2) The number of days required by 10 teams of workers to complete 10 installation with the same characteristics has been: 21, 32, 15, 59, 60, 61, 64, 60, 71, y 80 days.

3) The price of a magnetothermic switch in 10 electricity business in a city are: 25, 25, 26, 24, 30, 25, 29, 28, 26, y 27 euro.

4) An urban minibus made 15 trips yesterday, transporting the number of passengers is indicated below: 13, 14, 15, 9, 5, 9, 2, 14, 10, 6, 10, 11, 13, 14 and 14.

Moments

The moments are the operators that unify the calculation of the measures of position, dispersion and shape, allowing to differentiate one distribution from another.

The theory of moments is applicable both for the distributions of a single variable and for the joint study of two or more variables.

The moments for a single variable can be differentiated into three classes:

A) **Respect to origin**. The origin is the variable.

$$a_r = \frac{\sum_{i=1}^{k} x_i^r n_i}{N}$$

for r =0, 1,2,… n_i = is the frequency in each element.

B) **Respect to the average.** The source is the average arimetica of the variable.

$$m_r = \frac{\sum_{i=1}^{k} (x_i - \bar{x})^r n_i}{N}$$

Para r =0, 1,2,…

C) **Respect to any value of the variable.** The source is any value possible of the variable. Are little used and their calculation expression is the same with respect to the average, but substituting the value of the variable selected as origin.

$$M_r = \frac{\sum_{i=1}^{k} (x_i - k)^r n_i}{N}$$

Para r =0, 1,2,…

Example 1:

With the information presented, it calculates what is requested:
The following information is presented about the sales made of a product that are: 2, 3, 7, 8 and 10. Calculate: a) First, b) Second and c) Third moment, considering it with respect to the origin.

a) First moment

$$a_r = \frac{\sum_{i=1}^{k} x_i^r n_i}{N}$$

$$a_1 = \frac{2+3+7+8+10}{5} = \frac{30}{5} = 6$$

b) Second moment

$$a_2 = \frac{2^2 + 3^2 + 7^2 + 8^2 + 10^2}{5} = \frac{226}{5} = 45.2$$

c) Third moment

$$a_3 = \frac{2^3 + 3^3 + 7^3 + 8^3 + 10^3}{5} = \frac{1890}{5} = 378$$

Example 2:

Now calculate of the sales of same product any, with respect to the average**.** Calculate:

a) First y b) Third moment

$$m_r = \frac{\sum_{i=1}^{k} (x_i - \bar{x})^r n_i}{N}$$

a) First moment $m_1 = \frac{(2-6)+(3-6)+(7-6)+(8-6)+(10-6)}{5} = \frac{0}{5} = 0$

b) Third moment $m_3 = \frac{(2-6)^3+(3-6)^3+(7-6)^3+(8-6)^3+(10-6)^3}{5} = \frac{-18}{5} = -3.6$

Activity 5. Individual from the values, determine the four moments with respect to the origin also consider the average in basis the examples:

a) 4, 7, 5, 9, 8, 3, 6

b) 13, 14, 15, 9, 5, 9, 2, 14, 10, 6, 10, 11, 13, 14 y 14

Closure:

Activity 6. Individual answer the questions in your notebook.

1) What are the measures of form?

2) A distribution is symmetric if

3) A symmetric distribution always has the form of

4) 4) An asymmetric distribution can be:

5) What are the aiming measures?

6) What is kurtosis?

7) How are the formulas of the coefficient of asymmetry and aiming with the moments represented?

Activity 7.- Solve the next problems.

1) The owner of a restaurant wants to say if the number of tables is enough. Therefore, so register the time that the customers occupy a table. Time in minutes of 10 customers 68.2, 69.7, 52.8, 69, 57.3, 52.8, 58.1, 69.7, 53.4.

a) Calculate the average, median and mode. la media, la mediana y la moda

b) Calculate the asymmetry and aiming coefficient.

c) Calculate the first and third moment with respect to the average.

UNIT II

Didactic Sequence Num. ___8______	**CORRELATION MEASURES**
Expected learning: Calculate the correlation measures and take desitions from the central tendency and its representation of data set.	
Skill to develop:	
Discipline:	M2 Make and solve math problems, by applying different approaches. M3 Interpret and explain the results by the math methods and contrast the established models or real situations. M4 Argue the problem solution, with numerical methods, graphics, antilithics or variational, by the language or mathematic language and its use in information and communication technologies. M6. Quantify, stage and contrast experimental or mathematically the space magnitudes and the physical properties of the objects that surround us. M7. Choose a determinist approach or one random for the phenomenon or process study and argue your relevance.
Generics:	1.1 Face the shown struggles and be conscious of its value, strengths and weaknesses. 1.6 Manage the available sources by counting the restrictions to reach succes, 4.1 Express your ideas and concepts by linguistics math or grafic representations. 4.5 Manage technology of information and comunication to gain information and express ideas. 5.3 Identify the systems, rules or core principles, that lie behind a phenomenon serie. 7.2 Identify the activities that are less or more difficult, by recognizing and controling its response in front of challenges or obstacles.
EXPECTED PRODUCT: From the data set information, find if there is a correlationship between the sets and if its this wat, find the regression line.	

Opening	**Development**	**Closure**
Activity 1. Exercise to determine the pendant or equation of the line. (Diagnostic)	**Activity 2.** Do a resume of the topic. (Heteroevaluation) **Activity 3.** Exercise (Coevaluation) **Activity 4.** Regresion exercise. (Coevaluation)	**Activity 5.** Closure exercises (Heteroevaluation)

OPENING:

Activity 1. In pairs make the next activity.

1) From the points (2, -3) and (4, 5) find:
   a) Pendant
   b) Straight line ecuation.
   c) Graphic.
2) With the professor's help, answer the following questions:

How is the obtained pendant with the given points?

What is the inclination of the straight?

DEVELOPMENT:

Activity 2. Individually, read the next topic and make a resume.

Correlation coefficient:

Correlation is the grade of relation between the variables and it determine how good a line equation describes or explains the relation between te variables

Is frequent that we study about the same poblation of two variable quantities that are statistically different, with the purpose to see the relation between them, that means, the changes in one of them affects the quantity of other. If this occurs, we say that the correlated variables or that exists correlation bewteen them.

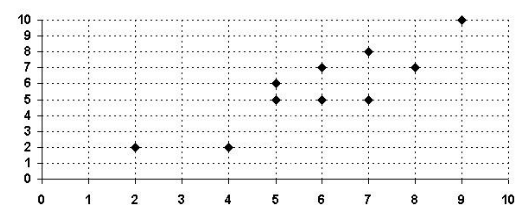
The analysis of the correlation implies the following steps:

- The descriptive study by the "scatter chart";
- The estimation of the correlation coefficient (including its confidence interval);
- The valuation of this correlation coefficient (sign and magnitude) and the statystic signification;
- The interpretation of the correlation coefficient by evaluating its determination coefficient;

First example.

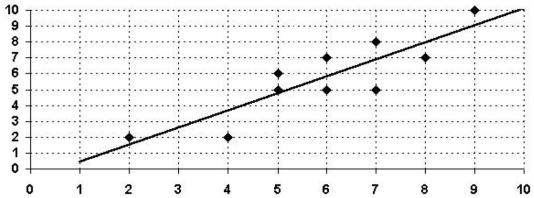
The students math and linguistics grades are being shown in the following table:

Math	2	4	5	5	6	6	7	7	8	9
Language	2	2	5	6	5	7	5	8	7	10



The values pares {(2,2), (4,2), (5,5),…;(8,7), (9,10)}, make a bidimensional distribution.

Cloud of dispersion diagram points: The first form to describe a bidimensonal distribution is to represent the value pairs in the Cartesian plane. The obtained graphic recives the name of point cloud or dispersion diagram



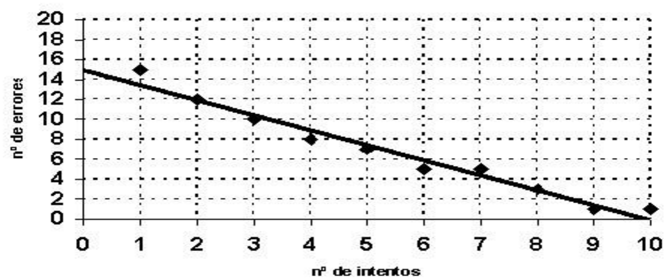
Lineal correlation and regression line: When we observe the point cloud, we can see that the points are grouped near a curve. Here, we will limit ourselves to see the points that are dispersing all around the line. If this occurs, we can say that there is lineal *correlation.* The line is denominated r*egression line.*

We will talk about the correlation line when the cloud is like the line and continuously is going to be weaker (Or less strong) when the cloud spread about the line.

On the graphic we can see that our correlation example is very strong, because the line that we are drawing is near the cloud points.

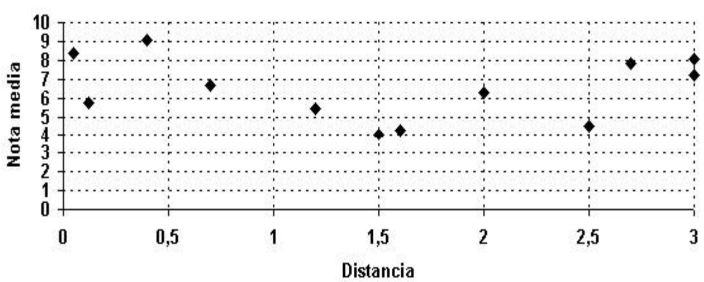
When the **line is increasing the correlation is positive or direct**: now when the variable is gaining, like in the last example. **When the line is decrescent the correlation is negative or inverse**; then the variable gain, the other one tends to lose.

Second example. A person trains to get the driver's license by repeating a test of 50 questions. The graph describes the number of errors that correspond to the attempts made. Note that there is a very strong correlation (the dots are "almost" aligned) and negative (the line is decreasing).



Third example. Twelve students from one center were asked how far away their residence was from the Institute, to study whether this variable was related to the average grade obtained. The data contained in the following table were obtained:

Distance (en km)	0,05	0,1	0,12	0,4	0,5	0,7	1	1,2	2,1	2,5	3	3
Medium note	8,4	4	5,7	9,1	6,3	6,7	4,3	5,4	7,8	4,5	7,2	8,1



We observe a cloud of points that does not suggest any specific line, because the correlation is practically non-existent, ie it has nothing to do with the academic performance the distance from the home to the Institute.

The most common linear correlation coefficients are: Pearson's and Spearman's, but we will only see the Pearson coefficient.

Parson's correlation coeficient

This type of correlation is applied for interval or reason variables and is calculated with the relation:

$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{(n-1)S_xS_y}$$

Sx = Standard deviation of X Sy = Standard deviation of Y

Relation coeficient characteristics.

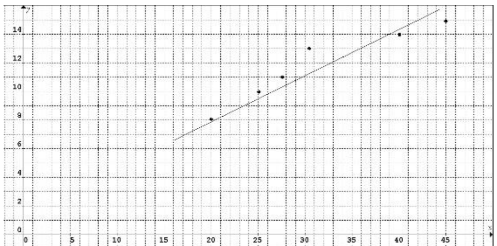
1. The correlation coefficient of the sample is identified by the lowercase letter **r.**
2. Displays the direction and strength of the linear (straight) relationship between two variables in range or reason scale.

2. It differs from -1 even of +1.
3. A near value of o indicates that teres a little association between the variables.
4. A near value of +1 indicates a direct or positive association between variables.
5. A near value of -1 indicates and inverse or negative association between variables.

Forth example. A political party had assorted the related information to their expenses and the number of political people obtained in different campaigns, because is interested in knowing effectively the assort on its blurb that has to do with the number of political people.

Assort in blurb (In millions of pesos) (X)	20	25	27.5	30.5	40	45
Number of political people (Y)	8	10	11	13	14	15

a) Drawing of the scatter diagram:



b) It calculates the measure of the variables X and Y.

$$\bar{X} = \frac{20+25+27.5+30.5+40+45}{6} = \frac{188}{6} = 31.33$$

$$\bar{Y} = \frac{8+10+11+13+14+15}{6} = \frac{71}{6} = 11.83$$

c) Calculate the correlation coefficient

X	Y	$(X - \bar{X})$	$(Y - \bar{Y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
20	8	2031.33=-11.33	811.83=-3.83	(-11.33)²=128.689	(-3.83)²=14.6689	43.3939
25	10	-6.33	40.11	-1.83	3.36	11.61
27.5	11	-3.83	14.69	-0.83	0.69	3.19
30.5	13	-0.83	0.69	1.17	1.36	-0.97
40	14	8.67	75.11	2.17	4.69	18.78
45	15	13.67	186.78	3.17	10.03	43.28
$\bar{X}$	$\bar{Y}$		445.83		34.83	119.33
31.33	11.83		89.17		6.97	$\sum (x - \bar{x})(y - \bar{y})$

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{(n-1)\, S_x S_y} = \frac{119.33}{(6-1)(9.44)(2.64)} = \frac{119.33}{124.61} = 0.957$$

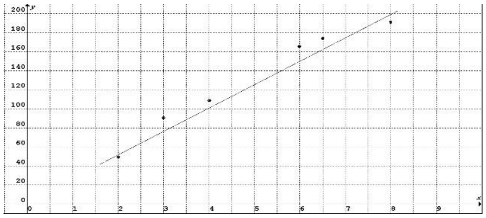
This indicates that there is a strong and positive correlation between variables, even though is smaller than one, manifests the presence of any other variable that is not included in the number of political people.

Fifth example. There are some programs to gain Reading speed in people, because of that company, decided to sell one of these systems, so it took a teenager that was studying the university and, for 8 weeks, they observe the number of words that he could read in a minute.
How can we know that this program is efficient?

The information is:

Week	2	3	4	6	7	8
Speed	49	86	109	165	173	192

a) Drawing of the scatter diagram



b) It calculates the measure of the variables X and Y.

$$\overline{X} = \frac{2+3+4+6+7+8}{6} = \frac{30}{6} = 5$$

$$\overline{Y} = \frac{49+86+109+165+173+192}{6} = \frac{774}{6} = 129$$

a) Calculate the correlation coefficient.

2	49	2-5=-3	49-129=-80	$(-3)^2=9$	$(-80)^2=6400$	240
3	86	-2.00	4.00	-43.00	1849.00	86.00
4	109	-1.00	1.00	-20.00	400.00	20.00
6	165	1.00	1.00	36.00	1296.00	36.00
7	173	2.00	4.00	44.00	1936.00	88.00
8	192	3.00	9.00	63.00	3969.00	189.00
			28.00		15850.00	659.00
			5.60		3170.00	
		Sx	2.37	Sy	56.30	$\sum(x-x)(y-y)$

$$r = \frac{\sum(x-\overline{x})(y-\overline{y})}{(n-1)SxSy} = \frac{659}{(6-1)(2.37)(56.30)}$$

$$= \frac{659}{666.18} = 0.989$$

This indicates that there is a strong and positive correlation between variables, even though is less than one, the method that has been interpreted saying that the method of fast reading its more effective during the weeks.

Activity 3. In pairs make one of the following problems that are shown, by making a graphic of the scatter diagram and by determining the correlation coefficient to indicate if there is a lineal relation between variables.

1) Considering the next information, where “x” represents 10 different bancs and those bancs have a metropolitan area, and “y” represents the total fee of the escrow that the bancs have.

Bancs	198	186	116	89	120	109	28	58	34	31
Total quote	227	166	159	125	102	68	68	40	27	28

2) In the following information, presents the daily medium temperature in °F and the daily intaking that corresponds to the natural gas in cubic feet.

°F	50	45	40	38	32	40	55
$Feet^3$	25	50	62	74	83	47	18

3) The next chart indicates the daily medium temperature in °F in a period of 10 days, and the index of a medium bursatil (In 1998).

°F	63	72	76	70	71	65	70	74	68	61
Bursatil index	8385	8330	8325	8320	8330	8325	8280	8280	8300	8265

Regression line

The regression in the analysis of the relation between two variables, and above all, because, starting from this relation, can predict the future behavior of the dependant variable over the values of an independent variable.

The relation between the variables can adapt different forms: lineal, parabolic, exponential, but will only address the form a lineal equation.

The **lineal regression** can be understood as a technique that resumes the information in a doting cloud in a simple line.

If we have a bidimensional distribution and we represent a dotting cloud, the line that adjusts the better to this dotting cloud, reserves the name of regression line.

- The regression line of and above is represented as: $\hat{Y} = a + bX$ where:

$$b = r\left(\frac{Sy}{Sx}\right) \qquad a = \bar{Y} - b\bar{X}$$

First example: By having in mind the information of problem 4: a political party has ordered the relative information to expenses and the number of political people obtained in the campaigns, because is interested in effectively know the expenses from the blurb and if they have something to do with the number of political people.

Expenses in blurb (in millions of pesos) (X)	20	25	27.5	30.5	40	45
Number of political people (Y)	8	10	11	13	14	15

What would be the number of political people waiting if the expenses of the blurb were 35 million of pesos?
Solution:

To find the number of political people we must find the regression line: $\hat{Y} = a + bX$ so:

$$b = r\left(\frac{S_y}{S_x}\right) \quad bb = (00.999999)\frac{22.6666}{99.6666} = 00.226699$$

$$a = \bar{Y} - b\bar{X} = 11.83 - (0.267)(31.33) = 11.83 - 8.365 = 3.464$$

Therefore, the equation in the regression line is: $\hat{Y} = 3.464 + 0.267X$ so to 35 million of pesos, the prediction of the number of political people is of: $\hat{Y} = 3.464 + 0.267(35) = 12.809$ it means approximately 13 political people.

Activity 4. Answer each of the next problems in your notebook. Coevolution with a partner.

1) Find the regression line.

X	3	4	6	8	9	11	14
Y	2	4	4	5	7	7	8

2) Find the correlation coefficient and the regression line in the following information:

X	1	2	3
Y	4	3	1

CLOSURE:

Activity 5. Answer the problem that your professor indicates, to deliver, and make an heteroevaluation, the rest answer them in your notebook.

1) In a magazine or a newspaper search an article where is relation between variables and find:
a) The dispersion cloud of information.
b) The correlation coefficient of information.

2) An experiment has been made to study the relation between a dose of a stimulant and the time that a person reacts to that. The information is:

Doses (milligrams)	1	3	4	7	9	12	13	14
Reaction time. (seconds)	3.5	2.4	2.1	1.3	1.2	2.2	2.6	4.2

Calculate the correlation coefficient and present a scatter diagram.

3) A security company considers the vehicle number (y) that circulates in a certain highway to more than 120 km/h, can be depending on the number of accidents (x) that occur in it. For 5 days he obtained these results:

Accidents	5	7	2	1	9
Number of vehicles	15	18	10	8	20

a) Calculate the coefficient of the lineal correlation.

b) If yesterday there were 6 accidents, how many vehicles can we assume that they were circulating on the highway to more than 120 km/h?

c) Is this a successful prediction?

UNIT II

Didactic Sequence Num. __9______	SETTING THEORY
Expected Learning: Study the behavior that statistics offer for probability.	
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results of the results of the mathematical procedures and the controls with the established models or real situations. M4 Argument the solution is a problem, with numerical, graphical, analytical or variational methods, through verbal and mathematical language and the use of information and communication technologies. M6. It quantifies, represents and contrasts experimentally or mathematically the magnitudes of space and the physical properties of the surrounding objects. M7. Determine a determinant or random approach to the study of a process or phenomenon, and argue its relevance.
Generics:	1.1 Faces the difficulties that arise and is aware of their values, strengths and weaknesses. 1.6 Manage available resources taking into account the constraints to achieve your goals. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 4.5 Manages information and communication technologies to obtain information and express ideas. 5.3 Identify the systems and rules or core principles that underlie a series of phenomena. 7.2 Identify the activities that result in less and greater interest and difficulty, recognizing and controlling their reactions to challenges and obstacles.
EXPECTED PRODUCTS: Elaborate Venn diagrams to determine some operations between sets.	

	Opening	Development	Closure
	Activity 1: Rescue of previous knowledge (Self-evaluation)	**Activity 2:** Conceptual map **Activity 3:** Questions (Heteroevaluation) **Activity 4:** Questions **Activity 5:** Exercise of sets **Activity 6:** Problems with Venn diagrams (Coevaluation)	**Activity 7:** Exercise of columns (Co-evaluation) **Activity 8 and 9.** Exercises (Heteroevaluation)

OPENING:

Activity 1. Consider the states of the Republic and answer the following questions.

a) ¿Qué estados empiezan con la letra C?

b) Of those that start with the letter C, it indicates those that border on other countries.

c) Of the states of subsection b, which are northerners?

d) Of the states of subsection c, consider that which is adjacent to the state of Nuevo León.

Share with your classmates and teacher the information answered in this activity.

DEVELOPMENT:

Activity 2. Read the topic that is presented below and make a conceptual map of the different operations that can be done through sets.

Theory of sets

The idea of a set is basic in human thought. It is always convenient to group objects or things in order to classify or order. Intuitively we can say that set is something that has "elements or members"

For example:

- A collection of old coins.
- The members of the Senate form a group called the Senate of the Republic.
- The numbers 2, 3, 5; they form a set of three elements.

SET. It´s a group, class or collection of objects that have a common characteristic, where each of them is called the element of the set.

Methods to describe a set. There are three methods to describe a set:

1.- Verbal description of the elements: It is the simplest way to publicize the contents of a set. ***Examples:***

- "The set of numbers greater than 25"
- "The set of days of the week"
- "The set of current banknotes in Mexico".

2.- List of the elements:

These are separated by commas and enclosed in keys. This form allows to symbolically denote the content of a set.

Examples:

- "The set of whole numbers less than 10"; It can be represented by:

{1 2 3 4 5 6 7 8 9,,,,,,,,}

- "The set of vowels of the Castilian alphabet" can be represented by:

{a e i o u}

- Applying this method in the description of "The set of all numbers less than 1000"; we would have to write 999 numbers; however, it can be represented by:

{1, 2, 3, ..., 998,999}

• To represent infinite sets, for example "The set of all numbers greater than 5"; we would represent it by:

{6, 7, 8, ...}

It should be noted that we use the ellipses to give an idea of what are the elements that constitute it.

3.- Assembly constitution notation:
Given "The set of odd numbers greater than 4 and less than 14" that can be expressed by:
{x x / is a number mpar i greater than 4 and less than 14}

The symbology {| } that is called "set constitution notation" describes the set based on the conditions of an arbitrary group element, that is, it establishes the conditions under which any element may or may not belong to the set.

The braces {} indicate the set.
The vertical line "|" is read as ***"such that" the letter "x"*** is an arbitrary element of the set and in turn is a variable.

On the left side of the vertical line we read ***"the set of x"*** and on the right side of the vertical line we list the properties that characterize these elements.

Examples:

1) Verbal description of the elements "The set of all the elements less than or equal to 7"
List of the elements B = {1, 2, 3, 4, 5, 6,7}
Constitution of set B = {x / x is a natural number, less than or equal to 7}

2) Verbal description of the elements "The integers less than -2"
List of the elements B = {-3, -4, -5, -6, ...}
Constitution of set B = {x / x is an integer, less than 2-}

Activity 3. Answer each of the following questions.

1) Define that it is a set.
2) Mention three descriptive terms that could be used to "name" certain sets.
3) Write the elements of each of the following sets.
• The set of subjects that you are studying this semester.
• The set of the months of the year.
• The set of even numbers less than 25.
• The set of days of the week beginning with the letter M.
• The set of planets in the solar system.
4) Write the elements of each of the following sets described by the constitution notation.
B = {x / x is an odd number and less than or equal to 9} G = {x / x is a day of the week that starts with the letter L} 5) Describes each of the following sets by means of the notation of constitution. K = {October, November, December} G = {a, b, c, d, e, f}

Basic definitions of sets.

SET WELL DEFINED It´s said that a set is "well defined" when specifying which elements belong and which do not belong to the set.

Example: "Odd numbers that go from 5 to 15"; {5, 7, 9, 11, 13, 15}

Membership of elements in a set: To indicate "membership" the symbol "" ∈ is used.

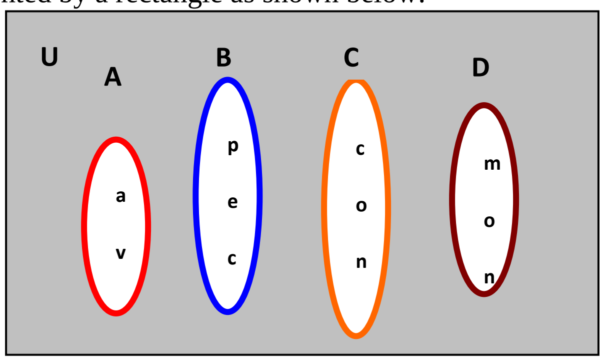
If "a" is an object and "A" is a set, we would write "a ∈A", which means that "a" is an element that belongs to the set "A".
The expression a∉ A indicates that the element "a" does not belong to the set "A".
Example: Given the set A = {a and i or u,,,,} to express that "u" is an element of set A, it is indicated as follows: "u A∈"

Given the set A = {a and i or u,,,,} to express that "m" is not an element of set A, it is indicated as follows: "m A∉"

UNIVERSAL SET	Is the set that contains the elements of the sets that are being considered in any analysis and is represented by the capital letter "U".

Examples:

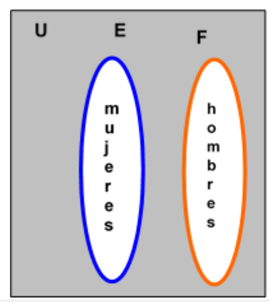
1) Let the sets A = {birds} B = {fish} C = {rabbits} D = {monkeys}
But there is another set that includes the sets A, B, C and D. It is U = {animals}.
Graphically, it is represented by a rectangle as shown below:



2) Be the sets $E=\{women\}$ $F=\{men\}$

But there is another set that includes sets E and F.

It is U = {human beings}. Graphically, it is represented by a rectangle as shown below:



EQUAL SETS.	Two sets A and B are equal when the first "A" contains the same elements as the second "B"; or vice versa.

In the equality of sets, the order of the elements does not matter.

If the set "A" is not equal to the set "B"; that is, they do not have exactly the same elements, we represent it by the expression A ≠ B; which reads "A different from B".

Examples:

1) Be the sets $A=\{b, g, o\}$ y $B=\{g, o, b\}$
It is established that they are equal because they contain exactly the same elements; therefore A = B.
2) Be the sets

$A=\{1, 2, 3, 4\}$ $C=\{1, 2, 3, 3, 4, 1\}$ $E=\{\text{vowel of the word world}\}$

$B=\{3, 4, 1, 2\}$ $D=\{1, 2, 2, 3, 4, 4\}$ $F=\{u, o\}$

Looking at each of the sets we can say:

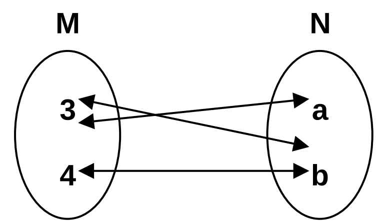
That the set A is equal to the set B that is A = B
That the set C is equal to the set D that is C = D That the
set E is equal to the set F that is E = F

CORRESPONDENCE ONE BY ONE	Two sets **M** and **N** have "One-to-one correspondence", if each element of **M** can be paired exactly with one of **N** and each element of **N** can be paired exactly with one of **M**.

Examples:

1) Let the sets M = {3 4 1,,} and N = {a b c,,}
In the sets there is one-to-one correspondence since each element M corresponds to only one of the set N.

Graphically represents:



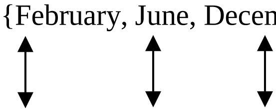
EQUIVALENTS SETS.	Two sets X and Y are "equivalent" when they have one-to-one correspondence.

Examples:

1) Let the sets X = {February, June, December} and Y = {Garza, Gómez, Mendez}

It is established that they are equivalent because they can be paired one by one, that is:

X = {February, June, December}



Y = {Garza, Gómez, Méndez}

SUB-SETS	When any set "A" in which all its elements are also members of another set "B", it is said that the set "A" is a subset of the set "B", the symbol used to indicate this relation is "" ⊂ .

Examples:

1) Let the sets K = {a, b, c, d, e} and L = {a, c, e} be
It is established that the set "L" is a subset of "K", symbolically represented by **L ⊂ K**.
How many possible sets of "L" can be set?
In response to the above, all possible sets of "L" will be:

A= { } A ⊂ L E = {a, c} E ⊂ L
A = {a} B ⊂ L F = {a, e} F ⊂ L
B = {c} C ⊂ L G = {c, e} G ⊂ L
C = {e} D ⊂ L H = {a, c, e} H ⊂ L o H

It is noted that the subset of "L" is the set of "H", which is exactly equal to the set "L", therefore, it states that *"any set is a subset of itself"*.

EMPTY SET	A set without elements is called an empty set and is represented by: Ø or {} It is also stated that *"the empty or null set is a subset of all the subsets".*

Examples:

$A = \{$ Dogs that fly $\}$	$A = \{\ \}$	A= Ø
$B = \{$x / x t's a month that has 53 días$\}$	$B = \{\ \}$	B= Ø
$C = \{$x / $x^3 = 8$ y x es impar$\}$	$C = \{\ \}$	C= Ø
$D = \{$x / x It's a 90-hour day $\}$	$D = \{\ \}$	D= Ø

POWER SET	If we are questioned, how many subsets can be obtained from a given set? in response, we have that the number of subsets resulting from any given set, are obtained by the expression T = 2, is where **"T"** is the number of subsets and **"n"** represents the number of elements of the given set.

Examples

1)	$M = \{1,2\}$ The set M has 2 elements.	Applying the formula T = 2n we can obtain how many Subsets are obtained from set M. $T = =2^2\ 4$	Therefore we obtain 4 subsets: $A = \{\}$ $B = \{\ \}1$ $C = \{\ \}2$ $D = \{1,2\}$
2)	$M = \{1,2,3\}$ The set M has 3 elements.	Applying the formula T = 2n we can obtain how many Subsets are obtained from Set M. $T = =2\ 8^3$	Therefore we obtain 8 subsets: $A = \{\}$ $B = \{\ \}1$$C = \{\ \}2$ $D = \{\ \}3$ $E = \{\ \}1,2$ $F = \{\ \}1,3$ $G = \{2,3\}$ $H = \{1,2,3\}$

SUBSET OWN	It´s established that the set **"A"** is a **"proper subset"** of the set **"B",** if all the elements of **"A"** are contained in **"B"** and if **"B"** contains at least one element not contained in **"A".**

Example:

1) Be the sets:

$A = \{$Monday, Wednesday, Friday $\}$ y $B = \{$Monday, Wednesday, Friday, Sunday$\}$

We note that all elements of set A are in set B, therefore **"A is a proper subset of B".**

INFINITY SET.-	There is an "infinity set", when it is not possible to indicate the number of elements that are contained in it.

Examples:

1) "The set of all natural numbers" $N = \{1, 2, 3, 4, 5, ...\}$

2) "The set of all even numbers" $M = \{2, 4, 6, 8, 10, ...\}$

FINITE SET	We have a "finite set", when it is possible to indicate the number of elements that are contained in it. Symbolically, the number of elements of a finite set is expressed by "n".

Examples:

1) If we have the set of days of the week, that is:

$K = \{$ Monday Tuesday Wednesday Thursday Friday Saturday Sunday $\}$

It is established that "K" is a finite set since it consists of 7 elements, that is: nK () = 7
It is necessary to clarify that a set can be finite, although it can be physically very difficult or out of human capacity, to determine how many elements are contained.

2) "The set of stars in the sky"
It is considered a finite set but who can tell?
3) M = {x x / is a river of the earth}
4) P = {x x / is a land country}

Activity 4. Correctly answer the following statements in your notebook.
1) Mention when you have a well-defined set.
2) Indicate if the following sets are defined and which are finite or infinite. {1, 4, 9, 16, 25, ...}
3) The group of mathematics teachers on your campus
4) {king, horse, level, ..., as}

Operations with sets

The operations with sets is the process that leads to form sets from other sets. The main joint operations are:

UNIÓ If **A** and **B** are two sets, then the union of A and B is the set formed by the **N** elements that are A or B or both and denoted **A**∪ **B**.

INTERSECTION. If A and B are two sets, then the intersection of A and B is the set consisting of the elements that are A and B simultaneously and is symbolized by A B∩
.

DIFFERENCE. From two sets **A** and **B**, another set is obtained, whose elements are those that belong to the set **"A"** but are not contained in the set **"B";** this process is called **"difference of sets or relative complement of B with respect to A"** and is represented symbolically by "**-**".

COMPLEMENT. From a set "A" and a set "U", another set is obtained, whose elements must be all those that are contained in the set "U" and that do not belong to the set "A", this process is called " *complement of any set in relation to a given universal set "*.
Symbolically, it is represented by a *"comilla"* that is located in the upper right part of the literal that defines any set.

CARTESIAN PRODUCT From two sets A and B, another set is obtained, whose elements are called

"Ordered pairs" that are written in curved brackets; the significant order of said ordered pairs, are indicated according to the position of the elements, that is, the first component belongs to the set "A" and the second component belongs to the set "b", this process is called **"Cartesian product"** and symbolically it is represented by "x".

Examples:

Make operations with sets in each of the following cases:

1) Be the sets $A = \{1, 2, 3, 4\}$ y $B = \{3, 4, 5, c, d\}$

Then the union of the sets is: $A \cup B = \{1, 2, 3, 4, 5, c, d\}$

2) Be the sets $S = \{a, b, c, d\}$ y $M = \{a, b, 1, 2, 3\}$

So the intersection of the sets is: $S M \cap = \{a\ b, \}$

Since "a, b" are the only elements that are both of S and M.

3) Be the sets $P = \{a, e, i, o, u\}$ y $Q = \{w, x, y, z\}$

It is observed that the sets do not have any element in common, so that their "intersection" is the "empty set"; that is to say: S M∩ = Ø Therefore, due to not having the same elements, the sets are said to be alien or different.

4) Be the sets $A = \{4, {}^{1}, 8, 10, 12\}$ y $B = \{10, 11, 12, 13, 14, 15\}$

The difference of these sets results in: $A\ B - = \{4,6,8\}$

5) Be $U = \{a, b, c, d, e, f, g, h, i, j\}$ y $S = \{a, g, h, i\}$ which is a subset of U.

So then $S' = \{b, c, d, e, f, j\}$

6) Be the sets $A = \{a\ b\ c, , \}$ y $B = \{c\ b, , s\}$

You can form another set that contains all the ordered pairs that result from the execution of the Cartesian product AxB, that is:

$$AxB = \{(a\ c\ a\ b\ a\ s\ b\ c\ b\ b\ b\ s\ c\ c\ c\ b\ b\ s,),(,),(,),(,),(,),(,),(,),(,),(,)\}$$

The sets that are used in the construction of a "Cartesian product" do not necessarily have to be different.

7) Be the set G= {2, 4, 6}, realizar la operación GxG.

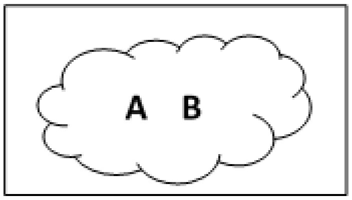
$$Gx\ G = \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$$

Venn diagrams

The easiest way to understand the ideas of set theory is by means of diagrams called **"Venn diagrams".** These graphs help us to relate between the sets to equality, the operations of union, intersection, difference and complement.

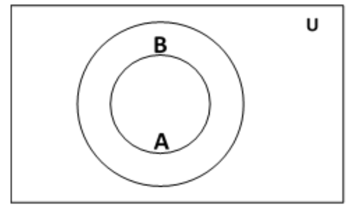
In the Venn diagrams, the sets are represented by ovules, circles or clouds and the point of U reference is the universal set "U" that is represented by a rectangle.

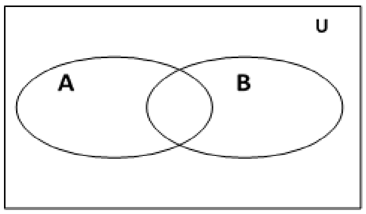
With the sets A and B contained in the universal set "U", by means of the Venn diagrams the following relationships can be determined:



In this diagram the equality between A and B is represented; it is also established that the set A is a subset from set B or vice versa. $\therefore A = B$, $A \subset B$ ó $B \subset A$

In this diagram, set **A** represents a subset of set **B**, that is, all elements of **A** are contained in **B**, while **B** has at least one element not contained in **A**.

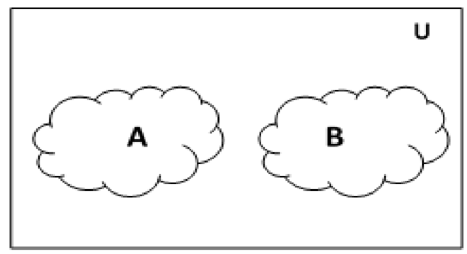




In this diagram the sets **A** and **B** have in common some, but not all the elements that is to say, represent the intersection between **A** and **B**

$\therefore A \cap B$

In this diagram, sets **A** and **B** do not have any element in common, that is, they represent two disjoint sets where their intersection is the empty set.



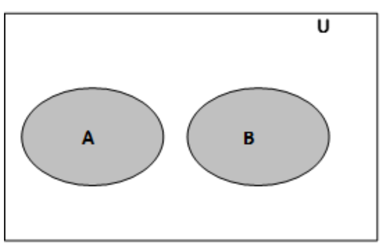
$\therefore A \cap B = \varnothing$

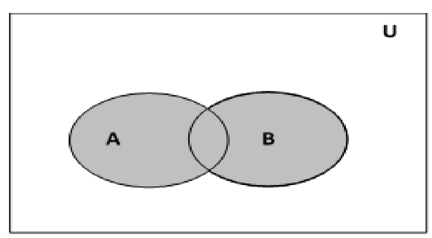
The main operations between two sets are represented by means of the following Venn diagrams. **The shaded surfaces** in the following figures **illustrate the union of set A with set B.**

$A \cup B$

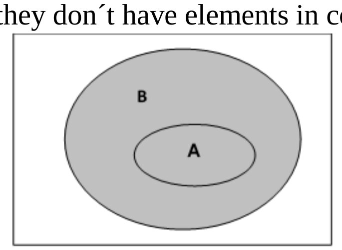
The union of sets A and B is the set formed by all the elements belonging to "A" or "B" or both.

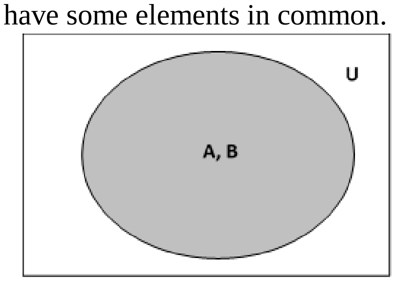
The union of sets is defined as: $A \cup B = \{x / x \in A ó x \in B\}$





When they don´t have elements in common When they have some elements in common.





When all the elements of a set The sets have exactly the same belong to another set Elements.

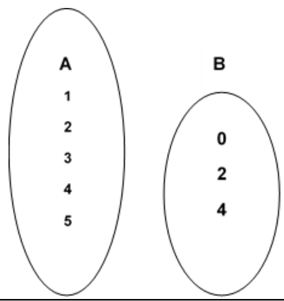
Examples:

1) Given the sets $A=\{0, 1, 2, 3, 4, 5\}$, $B=\{0, 2, 4\}$ y $C=\{5, 6, 8\}$ make and build the indicated diagrams:

a) $A\cup C$ b) $B\cup C$ c) $A\cup B$

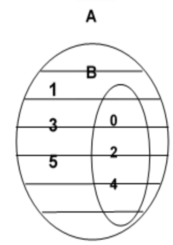
a) $A=\{0, 1, 2, 3, 4, 5\}$ y $B=\{0, 2, 4\}$

Graphic representation of the sets



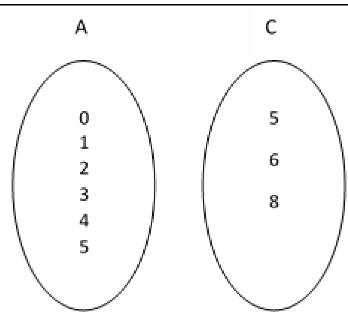
Graphical representation of the union of sets A and B.

$A\cup B=\{0, 1, 2, 3, 4, 5\}$



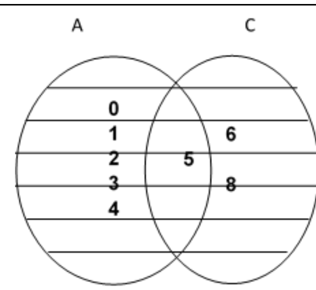
b) $A=\{0, 1, 2, 3, 4, 5\}$ y $C=\{5, 6, 8\}$

Graphic representation of the sets



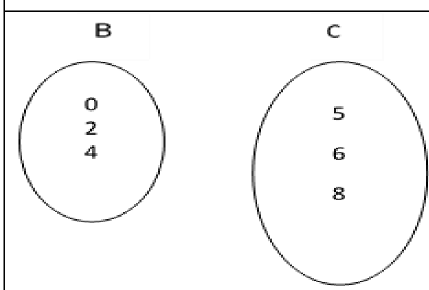
Graphical representation of the union of sets A and C.

$A\cup C=\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$



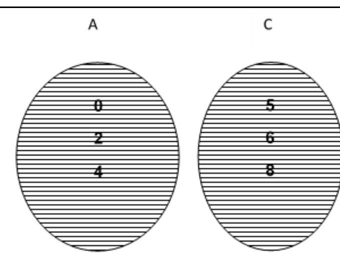
c) $B=\{0, 2, 4\}$ $C=\{5, 6, 8\}$

Graphic representation of the sets



Graphic representation of the union of sets B and C.

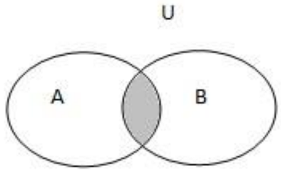
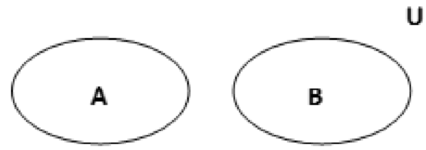
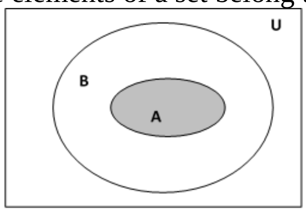
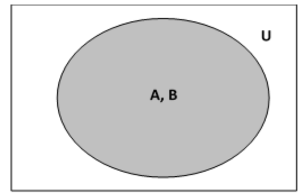
$B\cup C=\{0, 2, 4, 5, 6, 8\}$



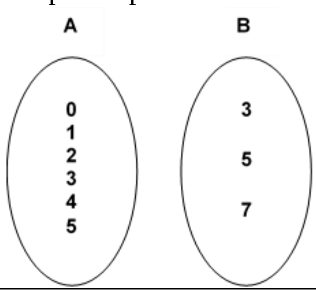
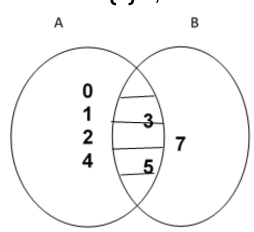
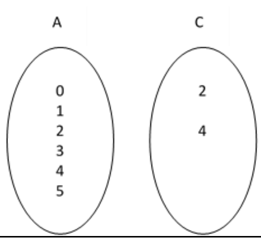
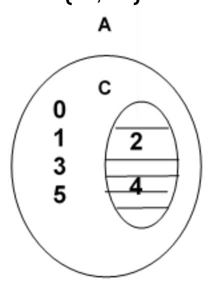
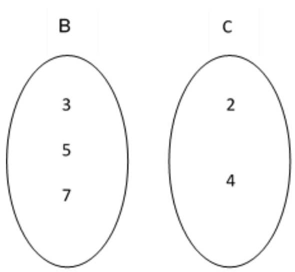
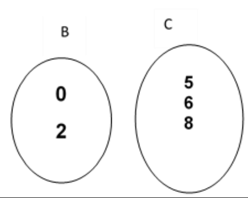
The shaded surfaces in the following figures, illustrate **the intersection of set A with the set B.** $A\cap B$.

The intersection of two sets A and B is defined to the set of elements that are common to **A** and **B**.

The intersection of sets is defined as: $A\cap B=\{x / x\in A \ y \ x\in B\}$

<table>
<tr><td>When they have elements in common

</td><td>When they don´t have elements in common

</td></tr>
<tr><td>When all the elements of a set belong to another set.

</td><td>The sets have exactly the same elements.

</td></tr>
</table>

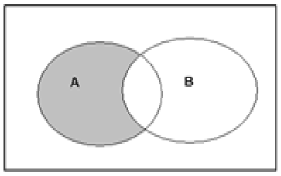
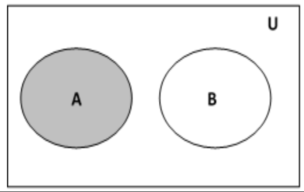
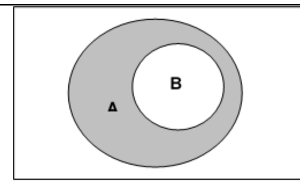
Examples:

<table>
<tr><td colspan="2">1) Given the sets A ={0, 1, 2, 3, 4, 5}, B ={3, 5, 7} y C ={2, 4} make and build the indicated diagrams:
a) $A \cap B$ b) $A \cap C$ c) $B \cap C$</td></tr>
<tr><td colspan="2">a) A ={0, 1, 2, 3, 4, 5}y B ={3, 5, 7}</td></tr>
<tr><td>Graphic representation of the sets

</td><td>Graphical representation of the intersection of the sets A y B. A ∩ =B { }3,5

</td></tr>
<tr><td>c) A ={0, 1, 2, 3, 4, 5} y C ={2, 4}
Graphic representation of the sets

</td><td>Graphical representation of the intersection of the sets A y C. A ∩C ={ 2, 4}

</td></tr>
<tr><td colspan="2">c) B ={3, 5, 7} y C ={2, 4}</td></tr>
<tr><td>Graphic representation of the sets

</td><td>Graphical representation of the intersection of the sets B y C.
B∩ =C Ø

</td></tr>
</table>

The shaded surfaces in the following figure illustrate **the difference between set A and set B**.

It is called difference of two sets A and B to the set formed by all the elements of A but that do not belong to B.

The difference of two sets is also defined as: $A - B = \{x / x \in A \text{ y } x \notin B\}$

When they have elements in common.	When they don´t have elements in common.	When all the elements of a set belong to another set.
		
		

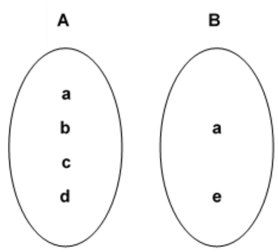
Examples:

1) Given the sets A = {a, b, c, d, e}, B = {a, e} and C = {d, f, g} make and construct the indicated diagrams:

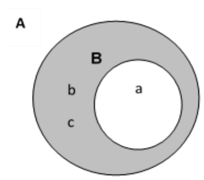
a) $A - B$ b) $B - C$ c) A-C

a) $A = \{a, b, c, d, e\}$ y $B = \{a, e\}$

Graphic representation of the sets Graphic representation of the difference of the sets A and B.

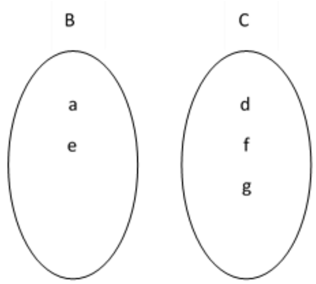


$A - B = \{b, c, d\}$



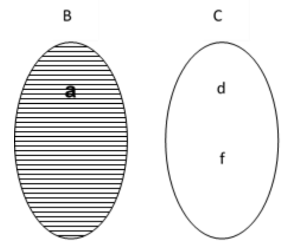
b) $B = \{a, e\}$ y $C = \{d, f, g\}$

Graphic representation of the sets Graphic representation of the difference of the



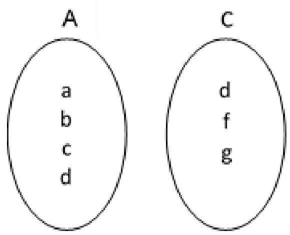
$B-C = \{a, e\}$

sets B and C.

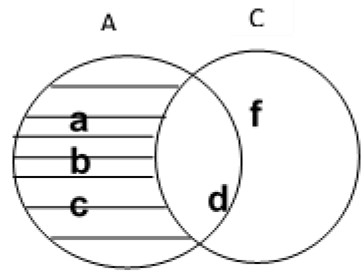


c) A ={a, b , c, d, e} y C ={d, f, g}

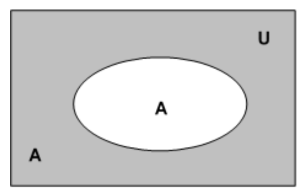
Graphic representation of the sets



Graphic representation of the difference between sets A and C. A -C ={a, b, c,e }



The complement A 'of the set A, is obtained by shading the surface of the universal set **U** not contained in **A**, that is to say:



If a set A is a subset of another universal set U, *to the set A* 'formed by all the elements of U but not of A, it is called the *complement of* A with respect to U. It is symbolically expressed:

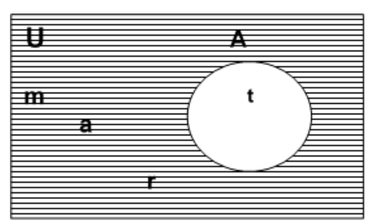
$$A\ x\ x\ U\ y\ x\ A' = \{\ /\in \quad \notin\ \}$$

Examples:

1) Be U ={m, a ,r, t, e} y A ={ }t, get the complement of A and build your diagram

The complement of A is: A'={m, a, r}

Graphical representation of the complement of A'={m, a, r}

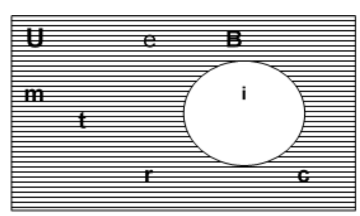


2) Be U ={letters of the word arithmetic} y B ={ vowels of the word life} get the complement of B and build your diagram U ={a, r,i, t, m, e, c} y B ={ }i, a

B'={r, t, m, e, c}

The complement of B is:

Graphical representation of the complement of B B'={r, t, m, e, c}



Activity 5

If we consider the sets: $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7\}$ y $C = \{2, 5, 6, 7\}$. Obtain what is indicated in each of the following cases and represent the results by means of the Venn diagram.

1) $A \cup C$	6) $A - C$
2) $A \cup B$	7) $A - B$
3) $B \cap A$	8) $C - B$
4) $A \cup B \cup C$	9) A'
5) $A \cap B \cap C$	10) B'

Application examples of Venn diagrams

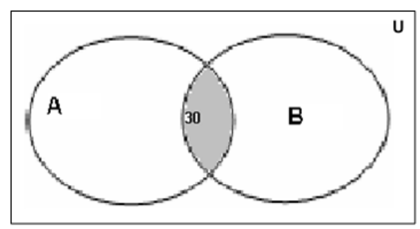
Example 1: In a language school there are 120 students of which 65 study German, 55 practice their English; 30 study both German and English at the same time. Applying the Venn diagram, determine:

a) Students who only study German

b) Students who only study English

c) The number of students studying German or English

d) The number of students who do not study any of these languages



Solution: We represent, through the Venn diagram, the universal set "U" of 120 students, the set "A" of students studying German, and the set "B" of students who study English; by the intersection of both sets, we have 30 students who study German and English at the same time, that is to say:

a) Students who only study German are: $n(A) - n(A \cap B) = 65 - 30 = 35$

Therefore, only 35 students study German

b) Students who only study English are: $n(B) - n(A \cap B) = 55 - 30 = 25$

Therefore, only 25 students study English

c) The number of students who study German or English are:

$$35 + 25 + n(A \cap B) = 35 + 25 + 30 = 90$$

d) The number of students who do not study any of these languages is:

$$U - \left[35 + 25 + n\,A(\cap B)\right] = 120 - 90 = 30$$

Example 2: A survey based on 250 high school students resulted in the following information about entering chemistry, physics and mathematics courses:

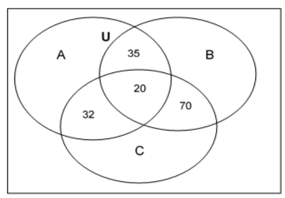
101 study chemistry
163 study physics
163 study mathematics
35 study chemistry and physics
32 study chemistry and mathematics
70 study physics and mathematics
20 study chemistry, physics and mathematics

a) How many students take chemistry as the only course?
b) How many do not follow any of the three courses?
c) How many study chemistry and mathematics, but not physics?
d) How many students do not follow the chemistry or physics courses?

Solution: We represent by means of the Venn diagram:

The universal set **"U"** is 250 students
The **"A"** set of students **studying chemistry**
The set **"B"** of students who **study physics**
The set **"C"** of students who **study mathematics**

By the intersection of the three sets, we have the 20 students who study chemistry, physics and mathematics at the same time.



a) Students who only take chemistry are: $n\,A\,n\,A\,B\,n\,A\,C\,A\,B\,C(\,) - (\cap -) \quad (\cap - \cap \cap) \left[\quad (\quad)\right] = 101 \;\; 35 - \; -$
$-32 \quad 20 = 14$

Therefore, only 14 students study only chemistry

b) Students who do not follow any of the three courses
First determine how many students take physics: n B () -n A (∩ -B) n B (∩ -C) n A [∩ ∩ (BC)] = 163 35
70 20 38- - - = You also determine how many students only take mathematics: $n\;C\;n\;A\;C\;n\;B\;C\;n$

$$A\;B\;C(\,) - (\cap -)\,(\cap -)\left[\;\cap\;\cap = (\;)\right]\; 163\;32\;70\;20 - - - = 41$$

Finally, it is determined how many students do not follow any of the three courses:

$$U - + + + \{14\;38\;41\;n\,A\,B\,n\,A\,C\,nB\,C\,nA\,B\,C(\cap +)(\cap +) \quad (\cap +)\;\left[\cap \cap(\;)\right]\} =$$
$$250\;14\;38\;41\;35\;32\;70\;20\;0 - - - - - - - - =$$

Therefore, all students follow at least one of the three courses

c) Los estudiantes que cursan química y matemáticas, pero no física, son:

$$14\;41 + +n\,A\,C(\cap = + + =)\;14\;41\;32 \qquad 87$$

Therefore, 87 students study chemistry and mathematics, but not physics

d) Students who do not study chemistry or physics, are: $14 + 38 - n(A \cap B) + 35 = 14 + 38 + 35 = 87$

Therefore, 87 students do not study chemistry or physics

Activity 6. Solve the following problems using Venn diagrams

1) In a group of 55 people there are only two types of individuals, economists and statesmen. If 35 are economics specialists and 31 are statesmen, how many are both statesmen and economics specialists?

2) An orchestra of musicians decides to form two musical groups, one of classical music and the other of a lounge, the first of 8 people and the second of 12 people; if three of the musicians belong to the two musical groups. How many members of the original orchestra did not decide to belong to any group?

3) The department of a maquiladora hires programmers, 25 of them for programming work of systems and 40 for applications programs, 10 of the contracted must perform both specialties. How many programmers should they hire?

CLOSURE:

Activity 7. Relate the columns by writing in the parentheses the number that corresponds to the correct answer.

1. Symbol that expresses the set that lacks elements.	( ) Equal sets
2. Two sets A and B are: ____ when the first "a" contains the same elements as the second "B" or vice versa.	( ) Finite set
3. It is the set whose elements can not be counted.	( ) Universal Set
4. It is a grouping, class or collection of abstract objects, where each of which is called the element of the.	( ) Infinity set
5. It exists when two sets have correspondence of elements one by one.	( ) Set
6. Set in which it is possible to indicate the number of elements that constitute it.	( ) Equivalent Set
7. It is the set that contains all the elements of the sets of a analysis any	( ) Ø o $\{\ \}$

Activity 8. Through the concept of sets develops the solution in each of the following cases.

1) Write the elements of each of the following sets.
   a) The group of groups of 3rd and 5th semester of your school.
   b) The set of the months of the year.
   c) The set of the five continents.

3) Write the elements of each of the following sets described by the constitution notation.
   a) A={x|x it's a two-wheeled tricycle }
   b) C={x|x is a president of the United Mexican States from 1994 to 2000}
   c) E={x|x is a figure of American cards }

4) Identify and mention if the following sets are defined or listed.

a {1, 4, 9, 16, 25, ….} ________________________.

b The set of all the students in the probability class whose last name begins with the letter "G" ________________________.

c The set of mathematics teachers on your campus ______________.

5) Mention whether or not the following sets are well defined.

a The set of world famous politicians. ___________.

b The set of divisible numbers between 3. ___________.

c The set of the directors of the schools of the CECYTE of Nuevo León. _______________.

6) Mark with an x the sets that are equal.

a) {a, e, i, o, u}, {u, a, o, e, i} ________

b) {letters of the Spanish alphabet}, {28} ________

c) {letters of the word LOVE}, {L, O, V, E} ________

d) {number of the colors of the rainbow}, {7} ________

Activity 9. Solve the following problems by applying set theory.

1.- Of 120 students, 60 study French, 50 Spanish and 20 study French and Spanish. Determine by means of a Venn diagram:

a) How many students only study French.

b) How many students only study Spanish.

c) The number of students who study French or Spanish.

d) The number of students who do not study any of these languages.

2. How many people were interviewed in a survey about television programs preferred by housewives, if they obtained the following information: 19 films, 23 concerts, 17 newscasts; Some people from these results added other preferences: 9 movies and concerts, 6 concerts and news, 4 films and news, 3 movies, concerts and news.

3. In a survey between workers and parents of a company, the following data were obtained: 775 have their own house, 800 car, 760 cable service; Of all these 300, they said that in addition to having their own home, they have a car, 250 houses and cable, 270 cars and cables and 200 in better economic conditions have the 3 things. How many have 2 things? and How many parents do not have any of these things?

UNIT II

Didactic Sequence Num. ___10____		**TREE DIAGRAM**
Expected Learning: Use counting or grouping techniques in the determination of probabilities by the tree diagram method.		
Competences to be developed:		
Discipline:	M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M6. It quantifies, represents and contrasts experimentally or mathematically the magnitudes of space and the physical properties of the objects that surround it. M7. Choose a deterministic or a random approach to the study of a process or phenomenon, and argue its relevance.	
Generics:	1.1 Faces the difficulties that arise and is aware of their values, strengths and weaknesses. 1.6 Manage available resources taking into account the constraints to achieve your goals. 4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 4.5 Manages information and communication technologies to obtain information and express ideas. 5.3 Identify the systems and rules or core principles that underlie a series of phenomena. 7.2 Identify the activities that result in less and greater interest and difficulty, recognizing and controlling their reactions to challenges and obstacles.	
EXPECTED PRODUCT: By means of a tree diagram, obtain the set of possible options when launching a coin 3 times.		
Opening	**Development**	**Closure**
Activity 1: questioning (Self-evaluation)	**Activity 2:** Conceptual map (Heteroevaluation) **Activity 3**. Exercises (Co-evaluation)	**Activity 4.** Closing exercises. (Heteroevaluation)

OPENING:

Activity 1. Answer the following questions in your notebook individually.

What are the characteristics of the trees?

What do you think of graphics? What is a chart?

DEVELOPMENT:

Activity 2. Make the reading of the theme and make a conceptual map in your notebook.

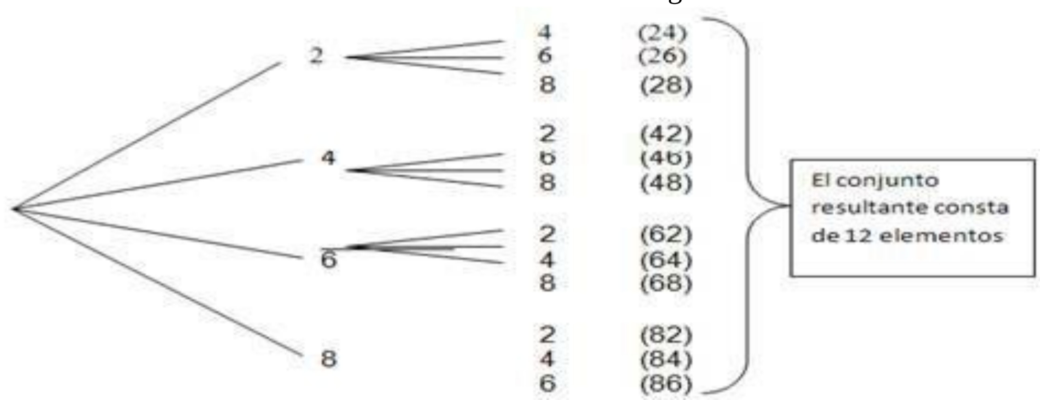
Tree diagram	It is a graph that illustrates how to list all the possible cases of a series of experiments, where each experiment can happen a finite number of ways

It is called a tree due to its appearance and is frequently used in connection with the fundamental principle of counting.

Example:

1) Determine how many numbers of two different figures can be formed with the following four digits: 2,4,6 and 8.

The resolution would be illustrated on a tree chart of the following:



Activity 3. Solve the exercises individually in your notebook.

1) Find the resulting set of the PxQxR product where:
   P= {a, b, c} Q= {1,3,5} y R= {d, e, f}

2) A committee of 3 members will be formed, composed of a representative of the workers, one from the administration and one from the government. If there are 3 candidates from the workers, two from the administration and 4 from the government, determine, how many different committees can be formed?

3**)** A binder offers two types of cover: hard or soft, and for each of them you can choose red, blue or green colors. In how many ways is it possible to bind a book?

CLOSURE:

Activity 4. In binas solves the following exercises, the final result design it on a flip chart and present it to your classmates, explaining how they solved it.

1) A coin is thrown three times. Represents the different possibilities that can happen.
2) A general practitioner classifies his patients according to: his sex (male or female), blood type (A, B, AB or O) and in terms of blood pressure (Normal, High or Low). Using a tree diagram say in How many classifications can this doctor's patients be?

UNIT II

Didactic Sequence Num. ___11__	**BINOMY THEORY AND PASCAL TRIANGLE**
Expected Learning: To know the concept of the theorem of the binomial and triangle of pascal for its application in the problems of everyday life	
Competences to be developed:	
Discipline:	M1 Construct and interpret mathematical models through the application of arithmetic, algebraic, geometric and variational procedures for the understanding and analysis of real, hypothetical or formal situations. M2 Formula and solve mathematical problems, applying different approaches. M3 Explains and interprets the results obtained through mathematical procedures and contrasts them with established models or real situations. M4 Argues the solution obtained from a problem, with numerical, graphical, analytical or variational methods, through verbal, mathematical language and the use of information and communication technologies. M8 Interpret tables, graphs, maps, diagrams and texts with mathematical and scientific symbols.
Generics:	4.1 Express ideas and concepts through linguistic, mathematical or graphic representations. 4.5 Manages information and communication technologies to obtain information and express ideas. 5.1 Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to the achievement of an objective. 5.3 Identify the systems and rules or core principles that underlie a series of phenomena. 5.4 Build hypotheses and design and apply models to prove their validity. 8.3 It assumes a constructive attitude, congruent with the knowledge and skills that it has within different work teams.

EXPECTED PRODUCTS: From the different ways in which a binomial can be developed to a power, develop some of them in an effective way.

Opening	Development		Closure	
	Activity 1: Questions (Self-evaluation)	**Activity 3.** Form (Heteroevaluation) **Activity 4.** Exercises **Activity 5:** Exercises (Co-evaluation)	**Activity 6:** Closing exercises (Co-evaluation)	

OPENING:

Activity 1. Answer the following questions in your notebook. In what semester do you remember that you heard the term "binomial"?

A binomial is a polynomial that consists of _________________, we also know that by definition the books

0 of Algebra handle that any number "a" raised to the potentiate matchjack__, estoesa = ___.

DEVELOPMENT:

Activity 3. Read the following information that will be very useful for the following activities in which you will have to solve exercises and make the form.

If we apply the previous conclusions to a general formula, it would be as follows:

Simplifying the previous general formula, we obtain the binomial theorem:

$$(x+y)^n = C(n,o)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + C(n,3)x^{n'-3}y^3 + \dots\dots\dots + C(n,n)y^n$$

If we remember the development of the first binomials, we have:

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Observe only the **COEFFICIENTS** of each binomial:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

This triangular arrangement formed is known as Pascal's triangle. As it can be seen that each row begins and ends with 1, that each next number after 1 forms its command the two numbers that are above it.

With the Pascal triangle and the binomial theorem we can find the development of any binomial or a specific term of the development of the given binomial.

Examples:

1) Calcula$(2x+3y)^6$

$$(2x+3y)^6 = C(6,0)(2x)^6 + C(6,1)(2x)^{6-1}(3y) + C(6,2)(2x)^{6-2}(3y)^2 + C(6,3)(2x)^{6-3}(3y)^3 + C(6,4)(2x)^{6-4}(3y)^4 + C(6,5)(2x)^{6-5}(3y)^5 + C(6,6)(3y)^6$$

$$(2x+3y)^6 = 1(64x^6) + 6(32x^5)(3y) + 15(16x^4)(9y^2) + 20(8x^3)(27y^3) + 15(4x^2)(81y^4) + 6(2x)(243y^5) + 1(729y^6)$$

$$(2x+3y)^6 = 64x^6 + 576x^5y + 2160x^4y^2 + 4320x^3y^3 + 4860x^2y^4 + 2916xy^5 + 729y^6$$

Activity 4. Individually develops the following binomials that are presented considering the formula that should be used for this and at the end presents the results obtained to the teacher for evaluation.

1) $(x+2y)^5$

2) $(3x+4y)^4$

3) $(x^2+y)^7$

Calculating only a specific term

The way to find a certain term without developing the binomial must make use of the formula that is presented below.

Each term is obtained with: $_nC_{r-1}\, x^{n-(r-1)} y^{r-1}$

Where **r** is the number of the place occupies a term in said development.

Example 1:

Get the third term of the binomial $(2x+3y)^6$

We know from the result of the previous development that the result is: $2160x^4y^2$ and
Consider the values of: n = 6 (since it is the exponent of the binomial) and r = 3 (corresponds to the term you want to find), substitute the formula presented, leaving the following expression.

$_nC_{r-1}X^{n-(r-1)}Y^{r-1} = {}_6C_{3-1}\,(2x)^{6-(3-1)}\,(3y)^{3-1}$

$= {}_6C_2\,(2x)^{6-(2)}\,(3y)^{2} = 15(16x^4)(9y^2) = \mathbf{2160x^4y^2}$

Activity 5. Find the indicated term in each of the following binomials, develop them.

1.- Get the 6th term of the binomial $(5x+4y)^8$

2.- Get the fourth term of the binomial $(a^2+b^3)^7$

3.- Get the last term of the binomial $(5x+3y)^5$

CLOSURE:

Activity 6. Individually and considering the examples presented, develop the following binomials or find the term that is requested depending on the case and in the end exchange the results obtained with a partner for its coevaluation.

1) Develop all the binomial $(5a+2b)^6$

2) Develop all the binomial $(a+b)^8$

3) Get only the third term of the binomial $(6x+7y)^9$

4) Get only the second term of the binomial $(3x+8y)^5$

UNIT III

Didactic Sequence Num. __12__	**COUNTING TECHNIQUES AND PRINCIPLES OF MULTIPLICATION AND ADDITION**

TRAINING INTENSIONS	
Purpose: To know the techniques that are used to enumerate different experiments difficult to quantify and apply the principles of addition and multiplication with real everyday examples.	
Competences to be developed:	
Disciplinary or Professional:	Formulates and solves mathematical problems, applying different approaches.
Generics:	**Develop innovations and propose solutions to problems based on established methods.** Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to achieving a goal.

Contents:

Fact (Know)	Procedural (know to do)	Attitudinal (know how to be)
Basic concepts and principles of addition and multiplication	The student will develop their activities individually and in a team of reading comprehension and identification of main ideas, as well as solving everyday problems of their environment where they apply the counting techniques and the principles of addition and multiplication in probability through an investigation. in the field to collect data on your campus to answer real questions.	The student will carry out their activities in a responsible manner and taking into account the indications presented; by socializing, respecting the contributions of colleagues and delivering the assigned tasks in a timely manner.

LEARNING PRODUCTS:

Opening	**Development**	**Closure**
Activity 1: Questions (Self-evaluation)	**Activity 2:** Form (Heteroevaluation) **Activity 3:** Exercises applying the principle of addition and multiplication (Coevaluation)	**Activity 4:** Questions (Heteroevaluation) **Activity 5:** Problems (Heteroevaluation)

OPENING:

Activity 1. Individually, answer the questions that are posed below.
Do you have any idea how the folios are organized to make the license plates of the cars?
How is it that the plates are not repeated and everyone has theirs?
Discuss these issues with your colleagues and note here the conclusions they reached.

DEVELOPMENT:

Activity 2. Carefully read the information presented below, without forgetting to highlight what you consider most important and make the corresponding form.

COUNTING TECHNIQUES Are those that are used to list events that are difficult to quantify.
The following questions are examples where you can make use of counting techniques.

1) How many clean-up committees of the institute can be formed if there are 150 students who want to help in this task and want to form commissions of eight students?

2) How many representations of students can be formed a) If you want them to consist only of Chemical Engineering students ?, b) Do you want the president to be a chemist ?, c) Do you want the president and treasurer to be chemicals? For all cases, it is desired that the representations consist of eleven students.

3) How many ways does a person have to select a washing machine, a blender and two blenders, if he finds 8 different models of washing machines in a store, 5 different models of blenders and 7 different models of blenders?

To determine without directly describing the number of possible cases of a particular experiment or the number of elements of a finite set, some basic principles are required to facilitate the process, highlighting:

- **The fundamental principle of counting**
- **Tree diagram**
- **The combinations**
- **Permutations**

Fundamental principle of counting

The two fundamental principles of basic counting are:

- **<u>Product rule</u>**: Used when a procedure is performed on separate tasks.
- **<u>Sum rule:</u>** Used when a procedure can be performed in several different ways.

Principle of multiplication: Also called "Fundamental principle of counting"

If an operation can be performed independently of **n1** different ways and if we continue the procedure a second operation can be performed independently of **n2** different ways after being performed, a third operation can be performed independently of **n3** ways

different, and so on up to nk in different ways. $n_1 \cdot n_2 \cdot n_3 \ldots n_k$

Where: **k** is the finite number of operations, then the total number of different ways in which all operations can be performed in the order indicated is the product of: $n_1 . n_2 . n_3 .... n_k$

Examples:

1) Determine how many numbers of two different figures can be formed with the following four digits: 2, 4, 6 and 8.

_____ _____
D **U**

Any of the 4 integers is chosen as the number of the tens (**D**), once chosen a digit, we have 3 integers from which we can choose the number of units (**U**).

When applying the fundamental principle of counting we have (4) (3) = 12. Therefore, **12 numbers of two figures** can be formed with the four digits given.

2) ¿Cuántos números de dos cifras podrán formarse con los cuatro enteros dados, si se permiten repeticiones?

Applying the fundamental principle of counting, the result is: (4) (4) = 16

(2,4)	(4,2)	(6,2)	(8,2)
(2,6)	(4,6)	(6,4)	(8,4)
(2,8)	(4,8)	(6,8)	(8,6)
(2,2)	(4,4)	(6,6)	(8,8)

Therefore, **16 two-digit numbers** can be formed with the four given integers, repeats being allowed.

Top of Sum

If one operation has n different results and another operation has m different results, different also to the results of the first operation and, in addition, if only one of the two operations can be carried out (that is, doing one does not allow the other to be done) , then the total of different ways of performing the joint operation (ie the first or the second) is n + m. The same applies to a larger number of operations.

Examples:

1) If a six-sided die or coin is thrown then the possible results are:

$$6 + 2 = 8$$

2) In a library there are 40 textbooks on discrete mathematics and 50 textbooks on calculus. Calculate how many different ways a student can choose a book from either of the two subjects.

$$50 + 40 = 90$$

Therefore, there are 90 ways in which a student can choose a book of either of the two subjects.

In addition, these two simple combined principles allow to obtain quite complex results.

3) A person wants to buy a clothes washer, for which he has thought that he can select from the brands Whirpool, Easy and General Electric, when he goes to make the purchase he finds that the washing machine of the W brand comes in two types of load (8 or 11 kilograms), in four different colors and can be automatic or semi-automatic, while the washing machine of the E mark, comes in three types of load (8, 11 or 15 kilograms), in two different colors and can be automatic or semi-automatic and the washing machine of the GE brand, is presented in only one type of load, which is 11 kilograms, two different colors and there is only semiautomatic. How many ways does this person have to buy the washing machine?

Solution: M = Number of ways to select a Whirpool washer

N = Number of ways to select an Easy brand washing machine

W = Number of ways to select a General Electric washing machine

The first step is to find the number of ways in which

M = 2 x 4 x 2 = 16 ways N = 3 x 2 x 2 = 12 ways W = 1 x 2 x 1 = 2 ways

M + N + W = 16 + 12 + 2 = 30 ways to select a washing machine

Activity 3. In pairs solves the following exercises applying the principle of addition and multiplication.

Top of the sum

1) A car spare part is sold in 6 Apodaca stores or in 8 Escobedo stores. How many ways can the spare be purchased?

2) It is desired to cross a river, for this there are 3 boats, 2 boats and 1 slider. How many ways can you cross the river using the means of transport indicated?

Principle of multiplication

3) In a car race 20 riders participate. Bearing in mind that it is not possible to arrive at the same time, in how many ways will the first three be able to reach the goal?

4) How many three-digit numbers can be formed with the digits 1,2, 3, 4, 5, 6, 7, 8 and 9 without repeating any number?

5) You want to change the flag of a city in such a way that it is formed by three horizontal stripes of equal width and different color. How many different flags can be formed with the seven colors of the rainbow?

6) To prepare a salad dressing a kitchen chef should choose a powder condiment, a type of oil and a type of vinegar. If you have 4 powder condiments, 3 types of oil and 5 types of vinegar, how many different dressings can you prepare?

CLOSURE:

Activity 4. Answer the following questions individually

1. Exemplify with a problem situation the fundamental principle of counting.
2. Mention the basic principles that facilitate counting techniques.

Activity 5. Solve each of the following problems by applying the principle of addition and multiplication or combined, as the case may be.

1) Juan is a distinguished high school student and the university offers him the careers of L.A.E., C.P., L.I. in the Faculty of Accounting. and L.N.I., the Faculty of Chemical Sciences offers Chemical Engineer, Industrial Engineer, Electronic Engineer and Computer Engineer and the Faculty of Humanities offers Communications, History, Philosophy and Literature; How many different study alternatives are offered to Juan?

2) Juan ex-student of the U.A.N. L. offer Cemex 3 different positions, in Alexa 2 different positions and in Neoris 4 different jobs; How many different work alternatives does Juan have?

3) How many different vehicle license plates can be constructed using two different letters of the alphabet followed by three digits, if the first digit can not be zero?

4) A man has 10 shirts, 7 pants and 5 ties in his wardrobe. How many ways can you choose a shirt, a tie and then a pair of trousers?

5) In the theater group they are doing tests for the Christmas work. Considering that 6 men and 8 women are presented for the main male and female roles. Calculate how many ways the director can distribute his main partner.

6) Juan decides to buy a car to move to his new job. Ford offers 3 different models and 2 forms of payment, in Volkswagen they offer 4 models and 3 forms of payment and in Nissan they offer 3 models and 3 forms of payment; How many different alternatives does Juan have?

7) An educational game contains figures in the shape of triangles, squares and circles, in two sizes, large and small, and in four colors, yellow, blue, red and green. How many different figures are there?

UNIT III

Didactic Sequence Num. ___13____		**FACTORIAL NOTATION**
TRAINING INTENSIONS		
Purpose: Apply the factorial concept in real problems of the environment through its development and using the calculator.		
Competences to be developed:		
Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.	
Generics:	**4. Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools.** Express ideas and concepts through linguistic, mathematical or graphic representations. Identify the key ideas in an oral text or speech and infer conclusions from them.	
Contents:		
Fact (know)	Procedural (know to do)	Attitudinal (know how to be)
Factorial Notation.	The student will develop his activities individually and in a team of reading comprehension and identification of main ideas, as well as applying his knowledge of factorial notation to solve everyday problems of his environment.	The student will carry out their activities in a responsible manner and taking into account the indications presented; by socializing, respecting the contributions of colleagues and delivering the assigned tasks in a timely manner.

LEARNING PRODUCTS:

Opening	**Development**	**Closure**
Activity 1: Questions rescue of knowledge (Self-evaluation)	**Activity 2:** Procedure of each example (Heteroevaluation) **Activity 3:** Problems (coevaluation)	**Activity 4:** Problems (Co-evaluation) (Heteroevaluation)

OPENING:

Activity 1. Individually answer the following:

What is a factor in mathematics? What mathematical operation talks about factors?

Take your calculator and look for the key that contains the symbol n! Maybe it is above a key, so you will use the second function or shift, type the number 8 and then shift and then the n key! What result did you get? ________

Now perform the next multiplication also by using the calculator.

(8)(7)(6)(5)(4)(3)(2)(1) = What happened?

DEVELOPMENT:

Activity 2. Carefully read the information presented below and carefully analyze the examples, writing the solution procedure of each example.

The symbol n !, which is read as a factor of n, represents the product of the n consecutive integers from 1 to n inclusive; mathematically expressed by:

$$n n n! = (-1)(n- 2)(n-3)....1$$

In the development of factorial quantities, it is always necessary to know the following equivalences that are defined by $0! 1= 1! 1=$

Examples:

1) Develop the factorial of 5

By the factorial symbol, it is denoted that n = 5 and applying the mathematical expression, we obtain:

5!= (5) (4) (3) (2) (1) 5! = 120

2) Calculate 7! $7! = 7\ 6\ (5)(4)(3)(2)(1)(\ )$ = 5040

3) Calculate $\frac{7!}{10!}$

Since $10!\ 10\ 9\ 8\ 7! = \bullet \bullet \bullet$ It has to $\frac{7!}{10!} = \frac{7!}{10\ 9\ 8\ 7! \bullet \bullet \bullet}$ so $\frac{7!}{10!} = \frac{1}{10 \bullet 9 \bullet 8} = \frac{1}{720}$

Activity 3.- Individually develop each of the problems that arise and then check the result on your calculator.

10! 7! 12! 6! $\frac{5!}{3!}$ $\frac{\quad}{(4\ 1)! -}$

CLOSURE:

Activity 4. Develop and solve each of the following problems involving factorial.

6! 10! $(6 -1)!$ $\frac{5!}{(3-1)!}$ $\frac{5!}{2(5-2)!}$

UNIT III

Didactic Sequence Num. ___14____		**PERMUTATIONS**
TRAINING INTENSIONS		
Purpose: Identify the accommodations of objects or people where order matters and the different situations in which permutations can be given.		
Competences to be developed:		
Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.	
Generics:	**8. Participate and collaborate effectively in diverse teams.** Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps.	
Contents:		
Fact (Know)	Procedural (know to do)	Attitudinal (know how to be)
Permutation.	The student will develop their activities individually and in a team of reading comprehension and identification of main ideas, analysis of examples, solution of permutation problems with real application of their environment and voluntary exposure of solutions to solve and explain a problem to the board.	The student will show solidarity by performing the activities assigned to work collaboratively and help their colleagues to work in a team with responsibility.

LEARNING PRODUCTS:

Opening	Development	Closure
Activity 1: Assignment of equipment (Heteroevaluation)	**Activity 2**: Form (Heteroevaluation) **Activity 3:** exercises of permutations (coevaluation) **Activity 4:** form (Heteroevaluation) **Activity 5:** Exercises of permutations (coevaluation) **Activity 6:** Completed form (heteroevaluation) **Activity 7:** Problems of circular permutations (coevaluation) **Activity 8:** Completed form (Heteroevaluation) **Activity 9:** Exercises (Co-evaluation) **Activity 10:** Completed form (Heteroevaluation) **Activity 11:** Exercises (Co-evaluation)	**Activity 12:** Conceptual map (Heteroevaluation) **Activity 13:** Exercises (coevaluation and hetero-evaluation)

OPENING:

Activity 1. Choose 3 classmates for the next activity and pass them to the front. Now form teams of 3 people with them. How many teams were formed? _______________

Now place 3 banks in front. They feel their companions. How many different ways can your peers sit down? _____________ What was the difference between the two activities?

To what it was that a different number came out?

DEVELOPMENT:

Activity 2. Carefully read the information presented below, carefully analyze the examples and make the theme form.

Permutation It is each of the possible ways in which the elements of a finite set can be ordered.

Example: If the given objects are three ***b***, ***g*** and ***o***, they are grouped taking the three at the same time, they can be arranged in the following ways:

bgo, bog, gbo, gob, obg, ogb | Ordered from left to right in six different ways.

Types of permutations

Permutations of n elements taken at the same time. The symbol nPn or P n n (,) is used, which represents the total number of all permutations of n different objects, taken from n to n, that is, n n =

Mathematically it is represented by the equation: $nPn\ n\ P\ n\ P\ n\ n\ n = !$ $\quad {}_{n}^{n} = !$ $\quad (,) = !$

In how many different ways can we put n different elements one at a time taken all at once and placing them in a row?

To give an answer, we consider that there are n empty spaces that must be occupied with n objects each space. The first space can be occupied in different ways, the second one (n - 1) different forms; the third of (n - 2) different ways and so on until we get to the last space that will be occupied by the only object we have left for the end.

Therefore the total number of possible permutations of the n elements would be: $nPn\ n$

$$n = (-1)(n- 2)(n-3)...1 = n!$$

Examples:

1) In how many different ways can 6 people be ordered in a row of 6 seats?

   The first seat can be occupied by any of the 6 people, that is, there are 6 ways to occupy the first seat. When the above has happened, there are 5 ways to occupy the second seat. Then there are 4 ways to occupy the third seat; 3 ways to occupy the fourth seat; 2 ways to occupy the fifth seat and only 1 to occupy the last seat.

Therefore the number of possible arrangements of the 6 people in a row of 6 seats, would be like this:

$$(6)(5)(4)(3)(2)(1) = = 6!720 \text{ Ways}$$

It can also be solved in the following way:

DATA	FORMULA	SUBSTITUTION	RESULT
$n = 6$ *persons* $n = 6$ *sits*	$_nP_n = n!$	$_6P_6 = 6!$ $= 720$	720 *Ways*

2) Five friends who are in a pool, after having been thrown by the giant slide, observe that each time they reach the top for the new release queue in different order. How many ways can you queue to throw yourself again?

Note that for the first position there are five people, four for the second, etc. In this way we have that the number of different ways to queue is:

DATA	FORMULA	SUBSTITUTION	RESULT
$n = 5$ *persons* $n = 6$ *postions*	$_nP_n = n!$	$_5P_5 = 5!$ $= 120$	120 ways

As we observed, in this case all the elements intervene and only the order of placement varies.

3) We want to exchange (fix) the letters **abc.** How many arrangements are obtained?

The arrangement of the letters would be as follows: **abc, acb, bac, bca, cab and cba.** There are 6 different permutations

DATA	FORMULA	SUBSTITUTION	RESULT
$n = 3$ *letters* $n = 3$ *positions*	$_nP_n = n!$	$_3P_3 = 3!$ $= 6$	6 arrangements.

Activity 3. Individually apply your knowledge to solve the following permutations.

1) In how many ways can 5 books be arranged on a shelf, if any arrangement is possible?
2) Get how many numbers can be formed with the digits 1, 2, 3, 4, 5 without repeating any digit
3) How many different ways are there to assign the starting positions of 8 cars that participate in a Formula One race? (Consider that the starting positions of the cars participating in the race are given completely at random)
4) A mother has 3 children. In how many different ways, naming them one by one, can you call them to dinner?
5) Seven men must be assigned to seven different jobs. How many ways can it be done?

Activity 4. Carefully read the information presented below, and make the corresponding form.

Permutation of n different taken in groups of r a permutations of n a time. The symbol nPr P n r (,) is used, which represents the total of elements different objects, taken from r to r, where r <n

Mathematically it is represented by the equation:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Examples:

1) Determine the number of permutations of the six integers 1, 2, 3, 4, 5, 6 taken from three in three

DATA	FORMULA	SUBSTITUTION	RESULT
$n=6$ $r=3$	${}_nP_r = \frac{n!}{(n-r)!}$	${}_6P_3 = \frac{6!}{(6-3)!}$ $= \frac{6!}{(3)!}$ $= \frac{6(5)(4)(3!)}{(3)!}$	${}_6P_3 = 120$ It´s the number of permutations of the 6 entires taken 3 by 3.

$7\ {}_3P =$

2) In a company five executives attend a meeting where there are seven chairs. Calculate how many ways the chairs can occupy

As only 5 chairs are occupied, the number of different ways of occupying them is equal to the number of permutations of 7 objects considered in groups of 5.

DATA	FORMULA	SUBSTITUTION	RESULT
$n = 7$ $r = 5$	${}_nP_r = \frac{n!}{(n-r)!}$ ${}_7P_5 =$	${}_7P_5 = \frac{7!}{(7-5)!}$ $= \frac{7!}{(2)!}$ ${}_7P_5 = \frac{7(6)(5)(4)(3)(2!)}{(2)!}$	${}_7P_5 = 2520$ Ways of occupy 5 chairs

Activity 5. In pairs solves the following problems of permutations with real application.

1) If 5 people get on a bus with 12 unoccupied seats, how many ways can they sit?
2) How many ways are there to assign the 5 playing positions of a basketball team, if the team consists of 12 members?
3) In how many ways can cespro.com place 3 system programmers in 3 different cities. If the programmers are available for any of 5 cities. Then you have 3 programmers available but there are 5 possible cities where they can go. How many ways could we locate them?
4) Let S = {Pérez, López, González, Moreno} from this set 2 people will be chosen for the positions of manager and supervisor, in how many ways can be done.

Activity 6. Carefully read the information presented below, without forgetting to highlight what you consider most important and complement the form.

Permutations circle It is defined as the possible arrangement of "n" objects around a or any circular other simple closed curve, where one of them maintains a fixed position. The number of circular permutations Pc of "n" objects is determined by the equation:

$$P_c = (n-1)!$$

Examples:

1) In how many ways can a meeting of 9 people be accommodated around a round table?

One person can sit in any fixed position of the table, the others can be arranged of 8! ways around the table, that is:

DATA	FORMULA	SUBSTITUTION	RESULT
n = 9	$P_c = (n-1)!$	$P_c = (9-1)!$ $P_c = (8)!$ $P_c = (8)(7)(6)(5)(4)(3)(2)(1)$	$P_c = 40320$

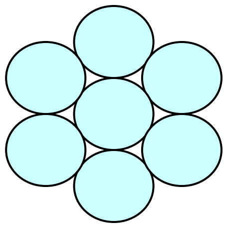
2) How many ways can 12 people sit at a round table if two insist on sitting next to each other? If we consider the two people who insist on sitting side by side as one; then there are six people to sit around the table and they can be accommodated from

DATA	FORMULA	SUBSTITUTION	RESULT
n = 6	$P_c = (n-1)!$	$P_c = (6-1)!$ $P_c = (5)!$ $P_c = (5)(4)(3)(2)(1)$	$P_c = 120$

But the two people considered as one can be ordered together from 2! ways In total there are:
$(5!)(2!) = (120)(2) = 240$

Therefore there are 240 ways to accommodate.

3) How many different ways can the figures from 1 to 7 be placed in the following figure?



This problem can be solved as the conjunction of two events: first I place a number in the center (7 possibilities) and second the other 6 figures, which by ordering in a circle can be exchanged for Pc = - (6 1)! shapes; Therefore:

The number of ways is equal to $(7)(5!) = (7)(120) = 840$

Activity 7. In pairs solves the following problems of circular permutations.

1) How many ways can they be accommodated in a meeting of 13 people at a round table?

2) Calculate how many ways you can sit 8 people around a table.

3) How many ways can 10 people sit at a round table?

Activity 8. Carefully read the information presented below, without forgetting to highlight what you consider most important and complement the form.

Permutations of objects that The number of different permutations of **"n"** objects, taken

are not all different. all at once, of which there are **"n1"** equal to each other, others **"n2"** equal

(Permutations with each other, etc. repetition).

Mathematically it is determined by the equation: $P = \frac{n!}{n_1!\, n_2!\, n_3! \ldots\ldots n_i!}$

Examples:

1) In how many ways can five red tiles be placed in a row, identical between 6 white tiles, also identical to each other and four blue tiles, equal to each other?

DATA	FORMULA	SUBSTITUTION	RESULT
$n = 15$ $n_1 = 5$ *red carts* $n_2 = 6$ *white carts* $n_3 = 4$ *blue carts*	$P = \frac{n!}{n_1!\, n_2!\, n_3! \ldots\ldots n_i!}$	$P = \frac{15!}{5!\,6!\,4!}$	$P = 630630$ Ways

2) Get all the possible signs that can be designed with six pennants, two of which are red, three are green and one purple.

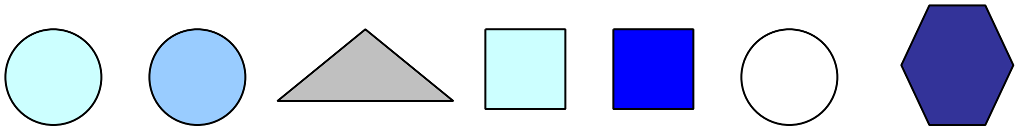
DATA	FORMULA	SUBSTITUTION	RESULT
$n = 6$	$P = \frac{n!}{n_1!\, n_2!\, n_3! \ldots\ldots n_i!}$	$P = \frac{6!}{2!\,3!\,1!}$	$P = 60$

$n_1 = 2$ red flags

$n_2 = 3$ green flags $\qquad n_1!\, n_2!\, n_3! \ldots\ldots n_i!$

$n_3 = 1$ purple flag

3) In how many different ways can the following figures be ordered?



DATA	FORMULA	SUBSTITUTION	RESULT
$n = 7$ $n_1 = 3$ circles $n_2 = 2$ *squares* $n_3 = 1$ triangle $n_4 = 1$ rhomb	$P = \frac{n!}{n_1!\, n_2!\, n_3! \ldots n_i!}$	$P = \frac{7!}{3!\, 2!\, 1!\, 1!}$	$P = 420$

Activity 9. Solve the next problems.

1) How many permutations can be made with the letters of the word CHIHUAHUA?
2) How many permutations can be made using all the letters of the word:
a) STATISTICS b) MATHEMATICS c) MISSISSIPPI
3) How many access codes to a computer will be possible to design with the numbers 1, 1, 1, 2, 3, 3, 3,3? 4) In how many ways is it possible to plant two walnuts, four apple trees and three plums on a dividing line?
5) How many different signs, each of 10 flags hung on a vertical line, can be formed with 4 identical yellow flags, 4 identical red flags and 2 identical blue flags?
6) They are ordered in a row 7 green balls, 4 yellow balls and 5 orange balls. If all the balls of the same color do not distinguish themselves, how many possible forms can be ordered?

Activity 10. Read the information carefully and continue to complete the form

Permutations with substitution. This permutation occurs when you have a number "n" of objects, we take one and before taking the other the one that had already been taken is replaced.

Mathematically it would be: $P_{cs} = (n)(n)(n)(n)\ldots(n) = n^r$

Permutaciones diferentes de tamaño r con sustitución (cs)

Examples:

1) Suppose that an urn contains 7 balls. Find the number of permutations that can be had by taking 3 one at a time, with substitution

Data	**Formula**	**Substitution**	**Results**
n = 7	$P_{cs} = (n)(n)(n)(n)\ldots(n) = n^r$	$P_{cs} = (7)(7)(7)$	$P_{cs} = 343$

1) How many ways are there to allocate three prizes for a draw where the first prize is a department, the second is a car and the third is a computer center? If the participants in this draw are 120 people and the allocation is can do with substitution.

Data	Formula	Substitution	Results
n = 120	$P_{cs} = (n)(n)(n)(n)\ldots(n) = n^r$	$P_{cs} = (120)(120)(120)$	$P_{cs} = 1,728,000$

Permutations without substitution This permutation is given when you have an "n" number of objects, we take one and to take the other one, the one that had already been taken is not replaced.

Mathematically it would be: $P_{ss} = n(n-1)(n-2)(n-3)\ldots(n-r+1) = \frac{n!}{(n-r)!} = {}_nP_r$

Examples:

1) In how many ways can you select a committee of three people from a group of 35 people, without substitution? Since the condition is that there is no substitution, then the first person can be selected in 35 different ways, the second person in the committee can be selected in 34 different ways, the third and last person can be selected in 33 ways, that is:

DATA	FORMULA	SUBSTITUTION	RESULT
n = 35 r = 3	$P_{ss} = \frac{n!}{(n-r)!}$	$P_{ss} = \frac{35!}{(35-3)!}$	$P_{ss} = 39270$

2) How many ways are there to be allocated three prizes of a draw where the first prize is a department, the second prize is a car and the third prize is a computer center?, if the participants in this draw are 120 people, and the allocation must be made without substitution.

DATA	FORMULA	SUBSTITUTION	RESULT
n = 120 r = 3	$P_{ss} = \frac{n!}{(n-r)!}$	$P_{ss} = \frac{120!}{(120-3)!}$	$P_{ss} = 1\ 685\ 040$

t should be noted that, in this case, as the tickets that are selected do not return to the ballot box from which they were drawn, the participants can only receive a prize if they were lucky. This is the way in which a draw is usually made.

Activity 11. As a team, solve each of the following problems by applying the formula of permutations with or without substitution as the case may be.

1) You have an urn with 10 balls. Find the number of permutations that can be obtained by taking 4, one by one:
a) With substitution b) Without Substitution

2) How many ways are there to assign the first five positions of a car race of formula K, if 26 cars participate in this race? Consider that the assignment is totally random. (Without substitution)

3) How many ways are there to assign the order of participation of the first 5 contestants of 11 finalists of a Miss World contest? (Without substitution)

4) How many three-digit numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 if the numbers can be repeated?

5) How many 4-digit numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, if the numbers can be repeated?

CLOSURE:

Activity 12. Individually, prepare a conceptual map classifying and defining each type of permutation, it must also contain the formulas and an example of each permutation.

Activity 13. Form teams of 3 or 4 people, identify the type of permutation to which each case belongs, as well as the formula to be used and resolved. Ahead!

1) Five people want to appoint a Steering Committee composed of a president, a vice president, a secretary, a treasurer and a member. How many ways can the committee be formed?

2) There are six flags of different colors. How many different signals can be sent using the six flags at the same time?

3) In a chess tournament 10 players participate, how many games will be played if each player plays against all the others?

4) If from a shelf we take 2 out of 3 books, how many permutations can be made?

5) How many ways can a president, a vice president, a secretary and a treasurer be selected from a group of 10 people?

6) We want to place 3 balls of red, blue and white in boxes numbered with 1, 2, ..., 10. We want to know the number of different ways in which the balls can be placed in boxes, if each box is capable to contain only one ball.

7) In how many ways can three tests be scheduled within a period of 5 days, so that the same day two tests are not scheduled?

8) Five people enter a room where there are 8 chairs. How many different ways can chairs occupy?

9) How many different four letter words can be formed with the letters LULU?

10) How many messages can be sent with ten flags using all of them, if they are four black, three green and three red?

11) How many different ways can eight people sit around a round table? 12) How many ways can we color an octagon with eight colors?

UNIT III

Didactic Sequence Num.___15____		**COMBINATIONS**	
TRAINING INTENSIONS			
Purpose: Identify the possible accommodations of objects or people where the order of them matters to apply the concepts and formulas of combination in the resolution of real problems.			
Competences to develop:			
Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.		
Generics:	**8. Participate and collaborate effectively in diverse teams**. Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps.		
Contents:			
Fact (know)	Procedural (know to do)		Attitudinal (know how to be)
Combinations	The student will develop their activities individually and in a team of reading comprehension and identification of main ideas, analysis of examples, solution of problems of combination with real application of their environment and exposure of the solutions to solve and explain the problems to the blackboard.		The student will practice respect by showing a positive attitude and recognizing the value of their classmates, their interests and their feelings.
LEARNING PRODUCTS			

Opening	**Development**	**Closure**
Activity 1: Rescue of previous knowledge (self-evaluation)	**Activity 2**: Summary (Heteroevaluation) **Activity 3:** Exercises in binas (coevaluation)	**Activity 4:** Closing exercises (Co-evaluation and hetero-evaluation)

OPENING:

Activity 1. Go back to the opening of sequence 3.4, remember the activities. If necessary, repeat it among your colleagues. Can you already distinguish a permutation from a combination?

Write down your differences or your comments on why it is easy to identify them

DEVELOPMENT:

Activity 2. Read the following topic carefully and summarize the topic.

Suppose you want to form a two-person committee that should be selected from a group of three people; How many committees can be formed?

Representing by (a, b, c) the group of 3 people, of which two will be taken to form the committee, **if we apply the formula of permutations** exactly 6 committees would be formed, being as follows:

(ab, ba, ac, ca, bc, cb)

We can say that we have 6 committees of size two, it is noted that **ab** and **ba** are different permutations because order is paramount in a permutation. But in combinations, **ab** and **ba** are indistinguishable, that is, the order in which the members of the committee are recorded have no influence on their integration.

Therefore, it is concluded that there can be only three committees of two people each, e96s to say, **(ab, ac, bc).** Each of the committees is called combinations where it is noted that order is not important in its elements.

"A combination is an unordered set of different objects"

COMBINATIONS "r" OF "n" OBJECTS: A combination of "n" objects taken from "r" is a selection of "r" from the "n" objects without attending to the ordering thereof

.

The number of "combinations" of "n" objects taken from "r" to "r" where r <n is symbolized by:

C(n,r) ó C ,rnnCr

Mathematically it is expressed by the equation: $^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{^{n}P_{n}}{r!}$

Examples: 1) Calculate 8C5

Data	**Formula**	**Substitution**	**Result**
n=8 r=5	$^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{^{n}P_{n}}{r!}$	$^{8}C_{5} = \frac{8!}{5!(8-5)!} = \frac{40320}{(120)(6)} = 56$	56

1) In a course of 15 men and 10 women In How many ways can a committee can be formed by 3 men and 2 women.

Data	**Formula**	**Substitution**	**Result**
n_1=15, r_1=3	$^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{^{n}P_{n}}{r!}$	$^{15}C_{3} = \frac{15!}{3!(15-3)!} = 455$	=(455)(45) =20,475
n_2=10, r_2=2		$^{10}C_{2} = \frac{10!}{2!(10-2)!} = 45$	

The committee can be formed 20475 ways

2) In an exam, a student must answer eight of a total of twelve questions, and must include exactly five of the first six. How many ways can your test be resolved?

Data	Formula	Substitution	Result

nr_{11}=5=6 $nn = \frac{nn!}{}$! $CC_{rr=5nn=65}$ $\overline{\quad}$!(6−56!)! = 6 $CC_{rr=3nn=63}$ $\overline{\quad}$!(6−36!)! = 20

=(6)(20) =120

CC_{rr} $(nn-rr)$

$rr!$

n_2=6 PP_{nnnn} r_2=3 =

$rr!$

The student can answer the test in 120 different ways.

Activity 3. In pairs solves the following problems of combinations with real application of the environment.

1) In how many ways can the wife of the principal of a school invite coffee to 3 of the 8 wives of the teachers of the school?
2) In a factory 9 men and 5 women apply to work. How many forms can the head of personnel make the selection if he can only hire 6 men and 3 women?
3) A research committee of 5 people will be organized between 7 representatives of the majority party and 6 of the minority party. Calculate the number of committees that can be formed if they must consist of exactly 3 representatives of the majority party.

CLOSURE:

Activity 4. As a team, as your teacher organizes, solve the following exercises and prepare an exposition so that they explain to the group how they solved these problems and what difference there is with the permutations.

1) In a high school, Mathematics students present an exam that includes 16 problems to solve 8 of them. How many different exams can you choose from those 16?
2) How many committees would be obtained from the previous problem if there are at least 3 of the minority party?

Didactic Sequence Num. 16	**CLASSICAL PROBABILITY**

TRAINING INTENSIONS		
Purpose: To know and apply the classic definition of Probability in the resolution of real environmental problems.		
Competences to develop:		
Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.	
Generics:	**Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools.** Express ideas and concepts through linguistic, mathematical or graphic representations. **8. Participate and collaborate effectively in diverse teams**. Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps.	
Contents:		
Fact (know)	Procedural (know to do)	Attitudinal (know how to be)
Classic probability.	The student will develop their activities individually and in a team of reading comprehension and identification of main ideas, analysis of examples, solution of probability problems with real application of their environment and voluntary exposure of solutions to solve and explain a problem to the board.	The student will practice the value of solidarity in carrying out the activities assigned to work collaboratively and help their colleagues to work in a team with responsibility.

PRODUCTOS DE APRENDIZAJE:

Opening	**Development**	**Closure**
Activity 1: Questions (Self-evaluation)	**Activity 2:** Summary (Heteroevaluation) **Activity 3:** problems (Co-evaluation)	**Activity 4:** Questions (Heteroevaluation) **Activity 5:** Problems (Coevaluation and Heteroevaluation)

OPENING:

Activity 1. Observe the following familiar scene and answer the questions that appear at the end of the image.

If the decision depends on the coin toss,

1.- Who do you think will help the father?

2.- Can you affirm with certainty who will have to wash the car?

3.- Describe in your own words, what you understand by "probability".

DEVELOPMENT:

Activity 2. Carefully read the information presented below, without forgetting to highlight what you consider most important and to make a summary.

Classical definition of probability

What is probability?

In the 17th century Blas Pascal, a French mathematician, took into account the previous question and also in response to a request made by a professional gambler, giving rise to an exchange of correspondence between Pascal and Pierre de Fermat, another French mathematician, who They discovered most of the most important concepts in probability theory. Although it began with gambling, probability theory has been gaining increasing importance until, at present, it contributes in an essential way to facilitating the studies of natural and social sciences and many of the practical problems of the world of business, industry and government.

The probability can also be found:

In the forecasts that are made about the result of something in which the action and the reaction of the humans intervene, as well as in situations that can not be affected by human actions like the daily reports of the atmospheric forecast, which indicate the probability that we have rain and / or cold.

The use of the concept of **"probability"** is related to the degree of confidence that the person has in which a particular event occurs.

The different types of probabilities to which we refer are based on:

I. The relative frequency.

II. The compound events. Axiomatic probability.

III. The conditional probability.

CLASSICAL DEFINITION OF PROBABILITY: If we assume that an event "E", where there are a total of "n" possible situations, all equally probable, of which one must occur, that is, can be presented in "S" situations.

Then the **probability "p"** that an **event occurs**, is given by the equation:

$$\text{Probability of the event} = \frac{\text{Number of ways the event can occur}}{\text{Number of possible results of the experiment}}$$

$$p = P\{E\} = \frac{S}{n}$$

The **probability** of non-occurrence or **non-occurrence "q"** of the event is given by the equation:

$$\text{Probability of the non-event} = \frac{\left(\text{Number of results possible of the experiment}\right) - \left(\text{Number of ways in what event may occur}\right)}{\text{Number of possible results of the experiment}}$$

$$q = P\{no\ E\} = \frac{n - S}{n} = 1 - \frac{S}{n} = 1 - p$$

From the above, we have to: $p + q = 1$ ó $P\{E\} + P\{noE\} = 1$ y $q = 1 - p$

The probability of the event **"p"** is also called **"probability of success",** and the probability of the event **"q" (**or also called event not **"p")** is also called **"probability of failure".**

NOTE: The probability obtained can be expressed as a percentage if we multiply the result by 100.

Examples:

1) What is the probability that a person of 25 years of age will survive until he is 40 years old, if according to a mortality table of every 93,745 people of 25 years of age, 87 426 reach 40 years of age?

Data	Formula	Substitution	Result
S = 87426 people who reach 40 years old n = 93745 total of people 25 years old	$P = \frac{S}{n}$	$P = \frac{87426}{93745}$	$P = 0.9325$

The result of the probability of the event in percent is given by: $(0.9325)(100) = 93.25\ \%$

2) In a box there are 25 screws in good condition and 80 defective. What is the probability of **removing a screw in good condition from the box at random?**

Data	Formula	Substitutionn	Result
S = 25 n = 80 + 25 = 105	$P = \frac{S}{n}$	$P = \frac{25}{105}$	$P = 0.2380$

The result of the probability of the event in percent is given by: $(0.2380)(100) = 23.8\ \%$

3) Of every 1000 people who have medical exams, 35 have vision problems.
What is the probability that an examinee suffers some discomfort with his sight?

Data	Formula	Substitution	Result
$S = 35$ $n = 1000$	$P = \frac{S}{n}$	$P = \frac{35}{1000}$	$P = 0035$

The result of the probability of the event in percent is given by: $(.0035)(100) = 3.5\%$

4) In a box there are 75 blue and 225 red marbles. What is the probability of drawing a blue marble at random?

Data	Formula	Substitution	Result
$S = 75$ $n = 75 + 225 = 300$	$P = \frac{S}{n}$	$P = \frac{75}{300}$	$P = 0.25$

The result of the probability of the event in percent is given by: $(.025)(100) = 25\%$

5) If we roll a die, what is the probability of getting an even number?

When throwing a die an even number of three ways can be obtained {2,4, 6}, of the six equally possible ways of obtaining {1, 2, 3, 4, 5,6}; so:

Data	Formula	Substitution
S={2,4,6,} (three) n={1,2,3,4,5,6}(six)	$p = \frac{S}{n}$	$p = \frac{3}{6} = \frac{1}{2}$

∴ **The probability of obtaining an even number is** ½

6) In a single die roll,

a) What is the probability of obtaining the numbers 1 or 6 ?;

b) What is the probability of not getting the numbers 1 or 6?

If the die is "well constructed", that is, it is not "loaded", there are six possible numerical cases to be presented {1, 2, 3, 4, 5,6}; since we are only interested in two results {1,6}; you have to: **a)**

Data	Formula	Substitution
S = {1,6} (dos) n ={1,2,3,4,5,6}(seis)	$p = \frac{S}{n}$	$p = \frac{2}{6} = \frac{1}{3}$

∴ **The probability of obtaining the numbers 1 ó 6 is 1/3 b)**

Data	Formula	Substitution
$p=\frac{1}{3}$	$q=1-p$	$q=1-\frac{1}{3}=\frac{2}{3}$

∴ **The probability of not getting the numbers 1 ó 6 is 2/3**

It is noted that **the probability of an event is a numerical value** between **0 and 1**; percentage between **0 and 100%**. If an event has to occur (possible), its probability is **1 or 100%.** If an event can not happen (impossible), its probability is **0**.

Activity 3. Solve the following problems individually and compare the results.
1.- When a die is thrown, what is the probability that it is an odd number other than 3?
2.- There are 52 cards numbered from 1 to 52. If a card is chosen at random, what is the probability that the respective number is a multiple of 12?
3.- A candy is extracted at random from a bag containing 4 chewing gum, 3 candies and 5 chocolates. Calculate the probability that you can get a chocolate, or a candy or chewing gum.

CLOSURE:
Activity 4. Answer the following questions individually and socialize your answers with the other classmates.
1) Describe in your own words what you understand by probability.
2) Currently, what are the applications of the word probability?
3) Cite the classic definition of probability.
4) Write the equation of the probability of an event.
5) Write the equation of the probability of failure.
6) What is the numerical value of the probability of an event?
7) What is the probability of a possible event?
8) What is the probability of an impossible event?

Activity 5. As a team, solve the following problems.
1) In a warehouse there are 3500 computers in good condition and 100 are defective.
What is the probability of choosing a computer in good condition?
2) Of every 1000 people who have medical examinations, 50 have vision problems. What is the probability that an examinee suffers some discomfort with his sight?
3) In a box there are 105 blue and 200 red marbles. What is the probability of drawing a red marble?
4) If we roll a die, what is the probability of getting an odd number?
5) In a single die roll,
a) What is the probability of obtaining the numbers 2 or 5 ?;
b) What is the probability of not getting the numbers 2 or 5?

UNIT III

Didactic Sequence Num. 17	**BASIC DEFINITIONS OF PROBABILITY AND SAMPLE SPACE**

TRAINING INTENSIONS

Purpose: To know and apply the definition of Probability as a relative frequency and to define sample space in the resolution of real environmental problems.

Competences to develop:

Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.
Generics:	. **Listen, interpret and issue pertinent messages in different contexts through the use of appropriate means, codes and tools.** Express ideas and concepts through linguistic, mathematical or graphic representations. Identify the key ideas in an oral text or speech and infer conclusions from them. **5. Develops innovations and proposes solutions to problems based on established methods.** Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to the achievement of an objective.

Contents:

Fact (Know)	Procedural (know to do)	Attitudinal (know how to be)
Basic definitions of probability.	Each student performs the corresponding activities for the development of meaningful learning of the subject.	The activities are intended to be carried out in order and with the necessary responsibility.

PRODUCTOS DE APRENDIZAJE:

Opening	**Development**	**Closure**
Activity 1: Team experiment (Self-evaluation)	**Activity 2:** Summary (Heteroevaluation)	**Activity 3**: Exercises (Co-evaluation and hetero-evaluation)

OPENING:

Activity 1. Students will perform the following experiment in teams:

1) Throw a dice 30 times and record in your notebook the numbers that appear in each throw
2) What observations does this experiment suggest?
3) In 30 runs it is expected that each value will come out, approximately _____. Compare it with the obtained values and comment if it is close to the expected value.
4) Complete the following table and create a definition of relative frequency in your own words.

Possible results	Frecuency (S)	Relative Frecuency Pf=S/n	Percent
1			
2			
3			
4			
5			
6			

DEVELOPMENT:

Activity 2. Read carefully the information presented below, without forgetting to highlight what you consider most important and make the corresponding summary.

Probability as relative frequency

Probability is the study of "random" or determination-free experiments. If a coin is thrown into the air, you have the certainty that it will fall, but it is not true to say that an "eagle" will appear. If we repeat the experiment of throwing the coin into the air; let "S" be the number of hits (number of times an "eagle" appears) and let "n" be the number of launches made, it is known then that according to the above, the "estimated or empirical probability" of a event is taken as the "relative frequency" of the occurrence of the event, when the number of observations is very large. The probability by itself is the "limit" of the relative frequency when the number of observations grows indefinitely. This gives rise to the problems of "verification" and "reliability", main topics of statistics.

The equation that defines **probability as relative frequency** is: $p_f = \frac{S}{n}$

Examples:

1) If in 100 tosses of a coin into the air, 53 "soles" result. What is the probability as a relative frequency?

Data	Formula	Substitution
S=53 *"soles"* n=100 launches	$p_f = \frac{S}{n}$	$p = \frac{53}{100} = 0.53$

∴ **The probability as relative frequency of "suns" is 0.53**

1) If in another 100 launches of the coin to the air, 49 "soles" result, considering the 100 launches described in the previous example, what is the probability as relative frequency in the total of the 200 launches?

Data	Formula	Substitution
S=(53+49) *"suns"* n=200 launches	$p_f = \frac{S}{n}$	$p = \frac{53+49}{200} = 0.51$

∴ **The probability as a relative frequency in the total of "suns" is 0.51**

If we continue with the experiment and according to the statistical definition, we will be getting closer and closer to the result of 0.5, which is the probability determined by the equation that classically defines the probability.

Basic Definitions

EXPERIMENT: It is a process that when repeated several times allows the observation of everything that can happen under certain circumstances. The results of an experiment give rise to a sample space S. The terms **"intent"** and **"test"** are synonymous with **"experiment",** since they refer to their execution.

If an experiment is done, it can have one of several possible outcomes; if it can not be predicted with certainty what will happen, **the experiment is said to be random**.

If an experiment has a single possible result, that when we do it we know that it will happen, **the experiment will be called deterministic.**

Random Experiment

If you throw a coin, whose result may be, fall eagle or fall sun. In this experiment we can not predict with certainty which result will appear with certainty.

When rolling a die, the results obtained can be any number from 1 to 6.

Deterministic experiment

Extract a ball from an urn containing balls with a single color, say black. If we look at the color of the extracted ball we know in advance that it is black.

SAMPLE SPACE: The set of all the possible results of an experiment, is called "sample space" of said experiment. The sample space is represented by "S".

A particular result, that is, an element of the set "S", is called "sample point" or "sample".

The sample space can be:

a) Finite: is one that is determined by a specific number of attempts, for example: throwing a coin four times in the air.

b) Infinity: that which is not determined by a specific number of attempts, that is, that these are "continuous" Examples:

When flipping a coin, the sample space is E = { come out face, come out stamp } ó E = {c, s}.

When launching a six-sided die, the sample space is:

E = { comes out 1, comes out 2, comes out 3, comes out 4, comes out 5, comes out 6} ó E = {1, 2, 3, 4, 5, 6} When throwing two coins, the sample space is: E = {(c,c), (c,s), (s,c), (s,s)}.

When throwing three coins, the sample space is E = {(c,c,c), (c,c,s), (c,s,c), (c,s,s), (s,c,c),(s,c,s), (s,s,c), (s,s,s)}

EVENT: It is a set of results, that is, it is any subset of a sample space. The "event" is represented by "E". The "event" consisting of a simple sample is called an "elementary event".

The empty set (Φ) and the sample space (S) are also events; the empty set (□) is sometimes called "impossible event" and the sample space (S) is called "true or safe event".

For example in the sample space E = {1, 2, 3, 4, 5, 6} of the roll of a die, the following are events:

1. Get a prime number A = {2, 3, 5}
2. Get a prime number and pair B = {2}
3. Obtain a number greater than or equal to 5 C = {5, 6}

1) A die is thrown on a table, point to the sampling points of the following events:

A: Exit a number 5.

B: Leave an odd number.
C: Exit a smaller number than 3.
D: Happen simultaneously A y B. E:
E: Happen simultaneously A y C.

Solution: A = {5} uno B = {1,3,5} C = {1,2} D= { 5} E =Φ Empty

2) 4 different currencies of $ 1, $ 5, $ 10 and $ 20 are launched; when it falls, it indicates how many sample points are and describes the sample space.

Solution
As each coin can fall in two different ways, we have: N = (2) (2) (2) (2) = 16 points
The sample space can be defined as ordered quadrants, where the first element is the result of the ONE peso coin, the second is $ 5, the third is $ 10 and the fourth is $ 20. Así:

	(a,a,a,a)					
	(s,a,a,a)	**(a,s,a,a)**	**(a,a,s,a)**	**(a,a,a,s)**		
S =	**(s,s,a,a)**	**(s,a,s,a)**	**(s,a,a,s)**	**(a,s,s,a)**	**(a,s,a,s)**	**(a,a,s,s)**
	(s,s,s,a)	**(s,s,a,s)**	**(s,a,s,s)**	**(a,s,s,s)**		
	(s,s,s,s)					

Observe: To find the elements of S, first it is considered that when throwing the coins do not leave sun, then write all possible events in which one of the coins comes out sun, then in which two suns come out, it is followed by all the events in which three suns come out and finally, it registers when four suns come out.

3) Define the sample space of simultaneous launch of a red and a blue die and determine the following probabilities:
a) The blue die is 4 o 6 b) The blue die is 4 or 6 and red is greater than 3.

Since each die has 6 faces and each result is formed of two colors, we have that the number of sample points is: N= (6 of the red die) (6 of the blue die) = 36 sample points.

The sample space can be defined by ordered pairs, where the first element is the result of the red die and the second is the result of the blue die:

	(1,1)	**(1,2)**	**(1,3)**	**(1,4)**	**(1,5)**	**(1,6)**
	(2,1)	**(2,2)**	**(2,3)**	**(2,4)**	**(2,5)**	**(2,6)**
S=	**(3,1)**	**(3,2)**	**(3,3)**	**(3,4)**	**(3,5)**	**(3,6)**
	(4,1)	**(4,2)**	**(4,3)**	**(4,4)**	**(4,5)**	**(4,6)**
	(5,1)	**(5,2)**	**(5,3)**	**(5,4)**	**(5,5)**	**(5,6)**
	(6,1)	**(6,2)**	**(6,3)**	**(6,4)**	**(6,5)**	**(6,6)**

a) From the sample space we can observe that there are six couples that contain the number 4 and six couples that contain the number 6.

Therefore the probability that the blue die is 4 or 6 is: $P = \frac{12}{36} \quad \frac{1}{3} =$

b) b) The pairs of the sample space where the blue die is 4 or 6 and the red one is greater than 3 are: (4,4), (5,4), (6,4), (4,6), (5,6), (6,6) therefore its probability is: $P = \frac{6}{36} \quad \frac{1}{6} =$

CLOSURE

Activity 3. In pairs solves the following problems and compares the result with your classmates. 1) If a person has a five peso bill, another ten and another twenty in his wallet. Describe the sample space of the experiment in which two bills are taken out of the portfolio, one after the other.

2) An experiment is to extract a marble from a box containing a mixture of red, yellow and green marbles. There are at least two marbles of each color.

a) List the sample space. b) If two marbles are removed from the box, list the sample space.

3) From a bag that contains 9 dice of red color and 6 dice of black color one dice is taken at random.

a) What is the sample space of the experiment?

b) What is the probability that the die removed is red?

4) Define the sample space of the simultaneous release of a red and a green dice and determine the following probabilities:

a) The green die is 3 or 6 b) The green die is 3 or 6, and red is greater than 2.

5) Describe the sample space for the experiment in which two coins are tossed simultaneously into the air, observing for each coin if eagle or sun falls and also determines the probability of obtaining:

a) At least one sun. b) Two eagles. c) The number of suns is equal to the number of eagles. d) Three eagles.

Didactic Sequence Num. ___18___	**AXIOMS AND PROBABILITY THEOREMS**

TRAINING INTENSIONS

Purpose: Apply theorems and axioms in obtaining the probability to solve everyday situations

Competences to develop:

Disciplinary or Professional:	Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches.
Generics:	**Develop innovations and propose solutions to problems based on established methods.** Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to the achievement of an objective. **Participate and collaborate effectively on diverse teams.** Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps.

Contents:

Fact (Know)	Procedural (know to do)	Attitudinal (know how to be)
Axioms and probability theorems.	The student will develop their activities individually and in binas of reading comprehension and identification of main ideas, analysis of examples, problem solving where the axioms are used and probability theorems with real application of their environment.	The student will practice respect by showing a positive attitude and recognizing the value of their classmates, their interests and their feelings.

LEARNING PRODUCTS:

Opening	**Development**	**Closure**
Activity 1: Questions (Self-evaluation)	**Activity 2:** Summary (Heteroevaluation) **Activity 3:** Exercise of relating columns (coevaluation) **Activity 4:** Exercises (Co-evaluation)	**Activity 5**: problems (Co-evaluation and hetero-evaluation)

OPENING:

A probability can not be any number, let's say -3 or 120%, if not a real number P, that is assigned to an event A that you have certain properties that we will see later.

Activity 1. Before starting the topic it is important that you read and answer individually the following questions, remembering what you saw in the topic Theory of sets.

1) What are mutually exclusive events?
2) What are complementary events?
3) How is the following notation A□B described?
4) How is the following notation A 'described?
Once you finish answering, compare the answers in a group.

DEVELOPMENT:

Activity 2. Below you will see some theorems and axioms of probability that will help you to obtain the probabilities of the different types of events that occur, rescue what you consider most important and analyze the examples in detail, summarize the topic.

Theorems and axioms of probability

Let "S" be a sample space of an experiment, let "E" be the class of events and let "P" be a function of real values defined in "E".

Therefore **"p"** is called **"probability function"** and **p (E)** is called the **"probability of event E"** if and only if the following are satisfied:

Theorems and axioms of probability

Let "S" be a sample space of an experiment, let "E" be the class of events and let "P" be a function of real values defined in "E".

Therefore **"p"** is called **"probability function"** and **p (E)** is called the **"probability of event E"** if and only if the following are satisfied:

AXIOMS

• If E is an event of S then the probability of event E is:

$$0 \le p(E) \le 1$$

- If the sample space S was obtained from a random experiment, then: $p(S) = 1$
- • If E1 and E2 are mutually exclusive events, then: $p(E_1 \cup E_2) = p(E_1) + p(E_2)$
- • If E1, E2, E3 ...... Ei are mutually exclusive and infinite events, then

$$p(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_i) = p(E_1) + p(E_2) + p(E_3) + \ldots + p(E_i)$$

• If E1, E2, E3 ...... In mutually exclusive and finite events, then:

$$p(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n) = p(E_1) + p(E_2) + p(E_3) + \ldots + p(E_n)$$

Now consider a series of theorems that are deduced directly from the previous axioms.

THEOREMS

The probability of an impossible event (∅) is equal to zero. **p(∅)=0**

If E 'is the complement of an event E, then: $p(E') = 1 - P(E)$

If E1 ⊂ E2, then: $p(E_1) \le p(E_2)$

If E1 and E2 are two events, then the probability of the difference of E1 relative to E2 is:

$$p(E_1 \cap E_2) = p(E_1) - p(E_1 \cap E_2)$$

If E1 and E2 are two events, then the probability of the union of two events is:

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

Corollary 1.- for events E1, E2 and E3, their probability of union is:

$$p(E_1 \cup E_2 \cup E_3) = p(E_1) + p(E_2) - p(E_1 \cap E_2) - p(E_1 \cap E_3) - p(E_2 \cap E_3) + p(E_1 \cap E_2 \cap E_3)$$

Examples:

1) From an urn containing 7 red tickets, 5 blue tickets and 6 green tickets, one is drawn at random. Find the probability that it is:

a) red b) blue c) green d) not red e) red or blue Solution:

a) Let E1 be the extraction event of a red ticket: E2 is the event of extracting a blue ticket and E3 is the extraction event of a green ticket.

.

Data	**Formula**	***Substitution***
S= 7 red tickets n = 7+5+6=18 total of tickets	$p(E_1) = \frac{S}{n}$	$P(E_1) = \frac{7}{18}$

∴ **The probability of extracting a red ticket is from 7/18**

b)

Data	**Formula**	***Substitution***
S = 5 blue tickets n =18 total of tickets	$p(E_2) = \frac{S}{n}$	$p(E_2) = \frac{5}{18}$

∴ **The probability of extracting a blue color ticket is 5/18** c)

Data	**Formula**	***Substitution***
S = 6 green tickets n =18 total of tickets	$p(E_3) = \frac{S}{n}$	$p(E_3) = \frac{1}{3}$

∴ **The probability of extracting a green color ticket is 1/3**

d) Let E1 'be the complementary event of E1 and refer to the non-extraction of a red ticket.

$$p(E_1') = 1 - p(E_1) = 1 - \frac{7}{18} = \frac{11}{18}$$

∴ **The probability of not extracting a red ticket is 11/18**

e) Are E1 "extraction of a red ticket" and E2 "extraction of a blue ticket", being mutually exclusive, so that:

$p(E_1) = \frac{7}{18}$

$p(E_1 \cup E_2) = p(E_1) + p(E_2)$

$p(E_1 \cup E_2) = 7$

+ 5 = 2 1818

3

$p\,E(_2) = \frac{5}{18}$

∴ The probability of not extracting a red or blue ticket is from 2/3

2) Of 150 students, 90 study mathematics, 80 study physics and 50 study mathematics and physics. If a student is selected at random, find the probability that it is:

a)A student of mathematics or physics   b) Do not study mathematics or physics

Solution:

Let the students be the students of mathematics, E2 students of physics and (E1∩ E2) students of mathematics and physics, being their probabilities:

a)

Data	Formula	*Substitution*
S= E_1= 90	$p(E_1) = \frac{S}{n}$	$P(E_1) = \frac{3}{5}$
S= E_2= 80	$p(E_2) = \frac{S}{n}$	$P(E_2) = \frac{8}{15}$
S= $E_1 \cap E_2$	$p(E_1 \cap E_2) = \frac{S}{n}$	$P(E_1 \cap E_2) = \frac{50}{150} = \frac{1}{3}$
n = 150		

Applying the equation for when E1 and E2 are not mutually exclusive, we have:

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2) = \frac{3}{5} + \frac{8}{15} - \frac{1}{3} = \frac{4}{5}$$

∴ The probability that he is a math or physics student is 4/5

b) Let p E (1'∩E2 ') = (E1 ∪ E2)' be the complementary event of (E1 ∪ E2) and it refers to not studying math or physics your probability is

:

$$p(E_1' \cup E_2') = p(E_1 \cup E_2)' = 1 - p(E_1 \cup E_2) = 1 - \frac{4}{5} = \frac{1}{5}$$

∴ The probability that he does not study mathematics or physics is 1/5

Activity 3. Instructions: Relate the following columns

( ) If the sample space S was obtained from an experiment, random then:	a) $0 \le p\,E(\ ) \le 1$
( ) If E is an event of S then the probability of event E is	b) $p\,E(_1 \cup E_2) = p\,E(_1) + p\,E(_2)$
( ) If E1 and E2 are two events, then the probability of the difference of E1 relative to E2 is:	c) $p\,E('') = -1\;p\,E(\ )$
( ) If E 'is the complement of an event E, then:	d) $p\,S(\ ) = 1$
( ) If E1 and E2 are mutually exclusive events, then	**e)** $p\,E(_1 \cap E_2) = p\,E(_1) - p\,E(_1 \cap E_2)$

Activity 4. Solve the following problems individually, at the end, exchange the present exercise with a partner to coevaluate.

1.- Two coins are thrown. What is the probability that at least one sun falls?

2.- In the CECyTE of Nuevo León, 35% of students failed math, 20% failed chemistry, and 10% both subjects.

3.- If a student is selected at random, what is the probability that he or she has failed any of the two?

CLOSURE:

Activity 5. In binas solves the following problems by applying axioms and probability theorems.

b) 1) Recent studies show that in a certain population of Mexico, the probability that an inhabitant is over 40 years old or has baldness is 0.40. The probability of being over 40 is 0.20 and the probability of having baldness is 0.30. Calculate the probability that an individual: a) Is 40 years old or younger.

c) b) Be older than 40 years without baldness.

d) c) Be older than 40 years with baldness.

2) Of 150 students, 80 play soccer, 70 play basketball and 50 play soccer and basketball. If a student is selected at random, find the probability that it is:

a) A student playing soccer or basketball

b) Do not play soccer or basketball

UNIT III

Didactic Sequence Num. ___19__	**CONDITIONAL PROBABILITY AND BAYES THEOREM**
TRAINING INTENSIONS	
Purpose Apply the principles and formulas of the conditional probability, as well as the Bayes theorem in the solution of real problems of daily life.	
Competences to develop:	
Disciplinary or Professional:	. Proposes, formulates, defines and solves different types of mathematical problems looking for different approaches
Generics:	**Develop innovations and propose solutions to problems based on established methods.** Follow instructions and procedures in a reflective manner, understanding how each of your steps contributes to the achievement of an objective. **Participate and collaborate effectively on diverse teams.** Proposes ways to solve a problem or develop a team project, defining a course of action with specific steps.
Contents:	

Fact (Know)	Procedural (know to do)	Attitudinal (know how to be)
Conditional probability, types of events and Bayes theorem.	The student will develop their activities individually and in a team of reading comprehension and identification of main ideas, analysis of examples, solution of problems of conditional probability and Bayes theorem as well as giving an explanation to some students on a voluntary basis to the group of the solution of problems with real application of your environment	The student will practice the value of solidarity in carrying out the activities assigned to work collaboratively and help their colleagues to work in a team with responsibility.

LEARNING PRODUCTS:

Opening	Development	Closure
Activity 1: Questions (Self-evaluation)	**Activity 2:** Summary Heteroevaluation) **Activity 3:** Questions (Co-evaluation) **Activity 4:** Problems (Co-evaluation)	**Activity 5:** Exercises (Heteroevaluation) **Activity 6:** Exercises (Heteroevaluation)

OPENING:

Activity 1. Analyze the following situation and answer the question:

If a coin of 1 peso and another one of 2 pesos is thrown, the fact that the weight falls eagle does not affect what happens when throwing the coin of 2 pesos.

What types of events are? Dependent or independent.

Once you have completed the information compare the answers with a partner.

DEVELOPMENT:

Activity 2. Individually, read the following information and make a summary.

Once you finish reading, write down in your notebook the concepts and formulas that you underlined and compare in a group.

Conditional probability

The conditional probability is applied in the calculation of an event when it is known that another event has occurred with which they are related; that is, the events are dependent. For example, calculate the probability of lowering the temperature in the Valley of Mexico if there is a "north" in the Gulf of Mexico.

Let E1 and E2 be two dependent events such that p (E1)> 0.

To express the probability of E2 given that E1 has occurred, p (E 2 / E1) is expressed.

Analogously if p (E2)> 0.

To indicate the probability of E1 given that E2 has occurred, it is expressed p(E1 / E 2)

Reduced sampling space

Considering the experiment of simultaneously launching a die and a coin once, the following sample space results:

S={1A, 2A, 3A, 4A, 5A, 6A, 1S, 2S, 3S, 4S, 5S, 6S }

And let the events E1 = {1A, 2A, 3A, 4A, 5A, 6A} let the coin fall eagle up and

E2 = {2A, 2S, 4A, 4S, 6A, 6S} that the die exhibits an even number upwards. If we ask the following question: What is the probability of event E2, knowing that event E1 has already occurred?

Answer: As the event E1 has already occurred, the only possible results are the six elements {1A, 2A, 3A, 4A, 5A, 6A} of said event. Of the six results, in three the die can appear with an even number on its upper face, so the probability of the event E2 is:

Data	Formula	Substitution
S = {2A, 4A, 6A}	p(E E /) = — S	p(E E /) 3) = = 0 5.
4A, n 6	2 1 2	1 n E= 1 = {1A, 2A, 3A,

Note: The symbol pE E (2/1) indicates the probability that event E2 will occur when the event has already occurred

E1

Therefore, the probability that the event E2 occurs when the event E1 has already occurred is 0.5.

Then to determine the probability of an event E2 knowing that the event E1 has already occurred is not considered as a sample space to S, but the event E1 (subset of S) is considered as the **reduced sample space**.

Examples:

1) Consider the experiment in which two dice are thrown, if the sum of the numbers that appear is greater than 6, what is the probability that one and only one of the dice falls with the 4 facing up?

Solution:

By the fundamental principle of counting, we have (6) (6) = 36 possible results of the experiment and consequently a sample space of 36 elements.

Let the event E1 be the sum of the numbers of the shot be greater than 6 and be the event E2 that one and only one of the dice fall with the 4 upwards, that is to say:

$$E_1 = \left\{ \begin{array}{c} (1,6),(2,5),(2,6),(3,4),(3,5),(3,6),(4,3),(4,4),(4,5),(4,6),(5,2) \\ (5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{array} \right\}$$

$$E_2 = \{(1,4),(2,4),(3,4),(4,1),(4,2),(4,3),(4,5),(4,6),(5,4),(6,4)\}$$

Considering that the event **E1** has already occurred, then the "reduced sample space" is **E1** and consists of 21 elements (simple or elementary events), of which 6 have one and only one of the dice with the 4 up.

The probability for the event (E2 / E1) is:

Data	Formula	Substitution
$S=\{(3,4),(4,3),(4,5),(4,6),(5,4),(6,4)\}$ $n(E_1) = 21$ Simple or elementary events.	$p(E_2/E_1) = \frac{S}{n}$	$p(E_2/E_1) = \frac{6}{21} = \frac{2}{7}$

Therefore, the probability of the event E2 when the event E1 has already occurred is of 2 / 7.

Probability of E2 given E1

Considering the experiment and the events of the previous example, obtain the probability of the event $E = E_1 \cap E_2$.

The event $E_1 \cap E_2 = \{(3,4),(4,3),(4,5),(4,6),(5,4),(6,4)\}$ contains six simple or elementary events and therefore its probability is:

Data	Formula	Substitution
$S = E_1 \cap E_2$ $n = 36$ Elements in total	$p(E_1 \cap E_2) = \frac{S}{n}$	$p(E_1 \cap E_2) = \frac{6}{36} = \frac{1}{6}$

Also the probability of event E1 is:

Data	Formula	Substitution

$S_{E_1} = 21$ elements $n = 36$ Elements in total	$p(E_1) = \frac{S}{n}$	$p(E_1) = \frac{21}{36} = \frac{7}{12}$

From the previous example, you know what $p(E_2/E_1) = \frac{2}{7}$ and relating these three results, we have to:

$$p(E_2/E_1) \cdot p(E_1) = p(E_1 \cap E_2)$$

The above leads to the following definition:

Let the events E1 and E2 of a sample space S have the property that the probability

$p(E_1) \neq 0$ and be $p(E_2/E_1)$ the probability of the event E2, since the event E1 has already occurred

The probability of pE E (2/1) is obtained by the equation:

$$p(E_2/E_1) = \frac{p(E_1 \cap E_2)}{p(E_1)} \text{ si } p(E_1) \neq 0$$

Examples:

1) Be the experiment of extracting two balls, one after the other, from an urn containing four red and three black balls. If E1 is the event "extract black ball is the first occasion" and E2 is the event "extract black ball is the second occasion". What is the probability that E1 and E2 occur?

Solution:

As we are asked for the probability of occurrence of the events E1 and E2, we will have: p (E1) is the probability of obtaining a black ball in the first extraction.

p (E2 / E1) is the probability of obtaining a black ball in the second extraction.

p (E1 ∩E2) is the probability that it will occur when dividing events E1 and E2.

Data	Formula	Substitution
$E_1 = 3$ elements $n = 7$	$p(E_1) = \frac{S}{n}$	$p(E_1) = \frac{3}{7}$
$E_2/E_1 = 2$ elements $n = 6$	$p(E_2/E_1) = \frac{S}{n}$	$p(E_2/E_1) = \frac{2}{6} = \frac{1}{3}$
$p(E_1 \cap E_2) = ?$	$p(E_2/E_1) = \frac{p(E_1 \cap E_2)}{p(E_1)}$ Clear $p(E_1 \cap E_2) = p(E_2/E_1) \cdot p(E_1)$	$p(E_1 \cap E_2) = \left(\frac{1}{3}\right)\left(\frac{3}{7}\right) = \frac{3}{21} = \frac{1}{7} = 0.1428$

Therefore the probability of E1 and E2 occurring is 14.28%

2) Three coins are thrown, what is the probability that they are all suns, and if the first of the coins is sun?

Solution:

A = is the event "the first coin is sun" (condition).

B = is the event "the three are suns".

2 → results of throwing a coin

3 → number of coins

$2^3 = 8$

$\{(SSS)(SAS)(SAA)(SSA)(AAA)(ASA)(AAS)(ASS)\}$

Data	Formula	Substitution
$A = \{(SAS)(SSA)(AAS)(ASS)\}$ n = 8	$p(A) = \frac{S}{n}$	$p(A) = \frac{4}{8} = \frac{1}{2}$
$B = \{(SSS)\}$	$p(B) = \frac{S}{n}$	$p(B) = \frac{1}{8}$
$A \cap B = \{(SSS)\}$	$p(A \cap B) = \frac{S}{n}$	$p(A \cap B) = \frac{1}{8}$
For what we have:	$p(B/A) = \frac{p(A \cap B)}{p(A)}$	$p(B/A) = \frac{1/8}{1/2} = \frac{2}{8} = \frac{1}{4} = 0.25$

Therefore, the probability of event B occurring as event A has occurred is 25%

Dependent events: When the occurrence of an event E1 affects the probability of occurrence of an event E2, then it is said that E1 and E2 are dependent events.

The probability of two dependent events is equal to the probability of an event multiplied by the conditional probability of the other, that is:

$p(E_1 \cap E_2) = p(E_1) \cdot p(E_2/E_1)$ y $p(E_1 \cap E_2) = p(E_2) \cdot p(E_1/E_2)$

1) A box contains 5 red chips and 4 blue chips. Let the event E1 "the first extracted record be blue" and the event E2 "the second extracted record be blue", in extractions without replacement. In this case the events E1 and E2 are dependent.

Solution:

Since the events E1 and E2 are dependent, their probabilities are:

Data	Formula	*Substitution*
S= 4 blue chips n = 5 + 4 = 9 total of chips	$p(E_1) = \frac{S}{n}$	$p(E_1) = \frac{4}{9}$

∴ **The probability that the first extracted card is blue is of 4/9**

The probability that the second extracted chip is blue, given that the first extraction was blue tab, is:

Data	Formula	*Substitution*
S= 3 blue chips n = 5 + 3 = 8 total of chips	$p(E_2/E_1) = \frac{S}{n}$	$p(E_2/E_1) = \frac{3}{8}$

∴ **The probability that the second extracted chip is blue is from 3/8**

The probability that both chips are blue, is:

$$p(E_1) = \frac{4}{9} \quad \text{and} \quad p(E_2/E_1) = \frac{3}{8}$$

$$p(E_1 \cap E_2) = p(E_1) \cdot p(E_2/E_1) = \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = \frac{1}{6}$$

∴ **The probability that both chips are blue is 1/6**

THEORY OF BAYES. It is an equation that allows to determine conditional probabilities. It applies to different events where at least one of them is known to have occurred.

Let E1, E2 and E3 be three different events of which at least one of them is known to have occurred. Suppose that with any of these, another event F may occur that is also known to have occurred.

If all the probabilities p E (1), p E (2), p E (3) and p F (/ E1), p F (/ E2), p F (/ E3) are known, we have the formula:

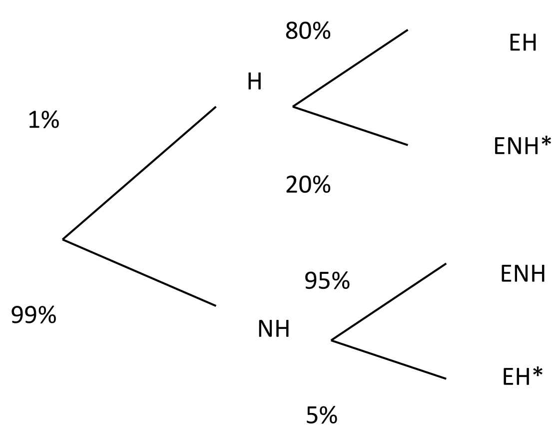
$$p(E_1/F) = \frac{p(E_1) \cdot p(F/E_1)}{p(E_1) \cdot p(F/E_1) + p(E_2) \cdot p(F/E_2) + p(E_3) \cdot p(F/E_3)}$$

Similar equations can be obtained for $p(E_2/F)$ y $p(E_3/F)$

"If an event can occur in more than one way, then the probability that it will occur in a particular way will be equal to the ratio of the probability that the form will occur with respect to the probability that it will occur."

Example 1: To determine if a person has "hepatitis" a blood test of a certain type is performed. The acceptance of this procedure is based on the following: among people "with hepatitis", 80% of the blood tests discover the disease; but 20% fail to do so. Among people "without hepatitis" 5% of the diagnoses indicate error as cases of hepatitis and 95% of the examinations indicate the correct diagnosis. If we take any one person from a group number of which 1% have hepatitis and that in a blood test shows that person has hepatitis. What is the probability that you really have the disease?

Solution:
By means of the tree diagram, we have:



H	people with hepatitis
NH	person does not have hepatitis
EH – ENH	correct hepatitis test
EH* - ENH*	exam with hepatitis error

Data

p(H) = 0.01
p(EH/H)= 0.8
p(NH)= 0.99
p(EH*/NH)=0.05

Formula

$$p(H/EH) = \frac{p(H)\,p(EH/H)}{p(H)\,P(EH/H) + p(NH)\,p(EH^*/NH)}$$

Substitution

$$p(H/EH) = \frac{(0.01)(0.8)}{(0.01)(0.8)+(0.99)(0.05)} = \frac{0.008}{0.0575} \approx 0.139$$

∴ **The probability that you really have hepatitis is approximately 13.9%**

2) Three X, Y and Z machines produce respectively 45%, 35%, and 20% of the total number of items from a toy factory. The hundreds of production defects of said machines are 2%, 4% and 6% respectively, if an item is selected at random, find the probability that the item is defective.

Solution: Let "A" be the event that an item is defective and applying the multiplication theorem for the conditional probability, we have:

Data

p(X) = 0.45 p(A / X)= 0.02 p(Y)=0.35
p(A / Y)= 0.04
p (Z)=0.2 p(A / Z)=0.06

Formula

$$p(A) = p(X)\cdot p(A/X) + p(Y)\cdot p(A/Y) + p(Z)\cdot p(A/Z)$$

Substitution

$$p(A) = (.045)\cdot p(.002) + p(.035)\cdot p(.004) + p(.02)\cdot p(.006)$$

$$p(A) = 0009 + 0014 + 0012.$$

$$p(A) = 0035.$$

∴ **The probability of the item being defective is 3.5 %**

3) Considering the information in the previous example, suppose that an article is selected at random and turns out to be defective. Find the probability that the item was produced by machine Y. Solution: It is about determining p (A / Y) by Bayes theorem, we have.

Data

p(X) = 0.45 p(A / X)= 0.02
p(Y)=0.35
p(A / Y)= 0.04
p (Z)=0.2 p(A / Z)=0.06

Formula

$$p(Y/A) = \frac{p(Y)\,p(A/Y)}{p(X)\,P(A/X) + p(Y)\,p(A/Y) + p(Z)\,p(A/Z)}$$

Substitution

$$p(Y/A) = \frac{p(.0\,35)\,(\,.p\;0\,04)}{p(\,.0\,45)\,(\,.P\;0\,02) + p(\,.0\,35)\,(\,.p\;0\,04) + p(\,.\,)\,(\,.0\,2\;p\;0\,06)} = 0\,4.$$

Therefore the probability that the article was produced by machine Y is from 40 %

Activity 3. Individually answer the following questions:

1.- It is applied in the calculation of an event when it is known that another event has occurred with which it is related; that is, the events are dependent.

2.- What type of events occur if the occurrence of one of them does not affect in any way the occurrence of the other?

3.- What type of events is when the occurrence of an event affects the probability of occurrence of another event?

4.- It is an equation that allows to determine conditional probabilities. It applies to different events where at least one of them is known to have occurred.

Activity 4. In binas solves the following problems of probability applying the concepts and formulas of conditional probability or Bayes theorem as appropriate.

1.- If 20% of the screws produced by a machine have a defect in the thickness, 10% have a length defect and 5% have both defects, if a screw is selected at random and it results with a defect in length. Is the probability that it also has a defect in the thickness?

2.- In a reading room there are 6 manuals of probability theory, 3 of which are in English.

The librarian takes 2 manuals randomly. Find the probability that both manuals result in English.

CLOSURE:

Activity 5. As a team solves the following problems of probability, when finished, go to the blackboard a member of each team to solve and explain the solution of the problem that the teacher indicates.

1) Consider the experiment of throwing two dice, if A is the event in the "first die an even number appears" and b is the event "in the second die the number 2 or 3 appears", what is the probability that it happens A and B?

2) The probability of a student failing Mathematics is 18%, of that he rejects Literature is 16%, that they fail both subjects is 4%. If a student is chosen at random and he failed Literature, what is the probability that he has also failed Mathematics?

3) Given the following probabilities: p H find p L H (/)

4) Two dice are thrown:
a) What is the probability of obtaining a sum of points equal to 7?
b) If the sum of points has been 7, what is the probability that one of the dice has left a three? Consider the events: A = "the sum of the points is 7" and B = "in one of the dice a three has come out".

5) A woman is a carrier of Duchenne's disease (muscular dystrophy).

Consider the following:
According to Mendel's laws, all possible genotypes of a child of a mother carrier (xX) and a normal father (XY) are xX, xY, XX, XY and have the same probability.

The sample space is W = {xX, xY, XX, XY} the event A = {ill child} corresponds to the genotype xY, therefore, the probability p (A) = 1/4 = 0.25 The event B = is defined {to be male} = {xY, XY}
The woman has the son and is male, what is the probability that she has the disease? **(the requested probability is P (A / B))**

5) It is known that 50% of the population smokes and that 10% smoke and is hypertensive. What is the probability that a smoker is hypertensive?

A = { be hypertensive }   B = { be a smoker } A∩B = { be hypertensive and smoker }

5) Two dice are thrown and it is known that the first one does not have the number 5. What is the probability that the sum of the dice is 8? P (A | B) To solve, let's call:

B the event: "the first die is not 5".   A is the event: "the sum of the dice is 8".

Activity 6. As a team, solve the following problems of probability. When finished, a member of each team will come to the board to solve and explain the solution of the problem that the teacher indicates.

1) A stationery has two ballot boxes, in the ballot box A there are 18 ballpoint pens of which seven are defective and in ballot box B, there are 22 ballpoint pens of which nine are defective. A pen is drawn at random from each urn. What is the probability that no ballpoint pen is defective?
2) A batch of 20 items have 10 defectives. Two items of the lot are chosen at random one after the other, what is the probability that they are not defective?
3) If we randomly select in succession two television kinescopes of a cargo of
240, of which, 15 are defective, what is the probability that both will be defective?
4) Determine the probability of randomly or randomly taking two aces from a deck of 52 cards.

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